Published by Institute of Physics Publishing for SISSA

Received: *October 31, 2007*
Revised: *January 13, 2008*
Accepted: *January 27, 2008*
Published: *February 11, 2008*

Giant gravitons - with strings attached (III)

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Abstract: We develop techniques to compute the one-loop anomalous dimensions of operators in the $\mathcal{N} = 4$ super Yang-Mills theory that are dual to open strings ending on boundstates of sphere giant gravitons. Our results, which are applicable to excitations involving an arbitrary number of open strings, generalize the single string results of hep-th/0701067. The open strings we consider carry angular momentum on an S^3 embedded in the S^5 of the $AdS_5 \times S^5$ background. The problem of computing the one loop anomalous dimensions is replaced with the problem of diagonalizing an interacting Cuntz oscillator Hamiltonian. Our Cuntz oscillator dynamics illustrates how the Chan-Paton factors for open strings propagating on multiple branes can arise dynamically.

Keywords: AdS-CFT Correspondence, Brane Dynamics in Gauge Theories, Gauge-gravity correspondence.

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1. Introduction and summary

The gauge theory/gravity correspondence [1 – 3] has provided powerful clues into quantum gravity. For example, the correspondence claims the exact identity of maximally supersymmetric $\mathcal{N} = 4$ Yang-Mills quantum field theory with gauge group SU(N) and type IIB string theory on the negatively curved $AdS_5 \times S^5$ space with N units of five form flux. Thus, we should be able to use the $\mathcal{N} = 4$ super Yang-Mills theory as a definition of quantum gravity on $AdS_5 \times S^5$. For interesting recent progress in this direction, see [4, 5]. The correspondence is however, not yet understood well enough, for this to be possible. A detailed understanding of the gauge theory/gravity correspondence is frustrated by the fact that it is a weak/strong coupling duality in the 't Hooft coupling. At weak 't Hooft coupling the field theory may be treated perturbatively, but the spacetime of the dual quantum gravity is highly curved. In the opposite limit of strong 't Hooft coupling we have to face the difficult problem of strongly coupled quantum field theory. The dual quantum gravity however, simplifies, because in this limit the curvature of the spacetime is small. For this reason, most computations which can be carried out on both sides of the correspondence (and hence clearly shed light on the correspondence) compute quantities that are protected by symmetry - typically supersymmetry (see [6] and references therein). The number of these tests and the agreement found is impressive. However, computing and comparing protected quantities is not satisfying - to probe dynamical features of the correspondence it would be nice to be able to compare quantities that are not protected by any symmetries. This is in general, a formidable problem. In [7], the notion of an *almost BPS state* was introduced. These states are systematically small deformations of states that are protected. For this reason, for almost BPS states, it is possible to reliably extrapolate from weak to strong coupling. A good example of almost BPS states are the BMN loops [8]. By studying BMN loops it has been possible to probe truly stringy aspects of the gauge theory/gravity correspondence (see [9] and references therein).

Giant gravitons, which are half-BPS states, have proved to be the source of many valuable quantities that are accessible on both sides of the correspondence. Further, they are very interesting from a string theory point of view, since they are good examples of protected non perturbative objects. Giant gravitons are spherical D3 branes extended in the sphere [10] or in the AdS space [11 – 13] of the AdS×S background. They are (classically) stable due to the presence of the five form flux which produces a force that exactly balances their tension. The dual description of giant gravitons is in terms of Schur polynomials in the Higgs fields [14, 15].

Our interest in giant gravitons is related to the fact that excited giant gravitons provide a rich source of nearly BPS states. Excitations of giant gravitons are obtained by attaching open strings to the giant. The gauge theory operator dual to an excited sphere giant is known and the anomalous dimension of this operator reproduces the expected open string spectrum [16].[1] This has been extended and the operators dual to an arbitrary system of excited giant gravitons is now known [20]. The dual operators, restricted Schur polynomi-

[1] See [17 – 19] for further studies of non-BPS excitations that have been interpreted as open strings attached to giant gravitons.

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als,[2] beautifully reproduce the restrictions imposed on excitations of the brane system by the Gauss law. Further, these excited giant gravitons have recently been identified as the microstates of near-extremal black holes in $AdS_5 \times S^5$ [21]. Although the evidence for identifying the restricted Schur polynomials as the operators dual to excited giant gravitons is convincing, much remains to be done. For example, we do not yet understand the detailed mechanism allowing Chan-Paton factors, expected for strings attached to a bound state of giant gravitons, to emerge from the super Yang-Mills theory. In this article, our goal is to explore this issue, by providing techniques which allow the computation of the anomalous dimensions of excited giant gravitons, to one loop. We will argue that the Chan-Paton factors emerge from the symmetric group labels of the restricted Schur polynomials.

The computation of anomalous dimensions of operators in $\mathcal{N} = 4$ super Yang-Mills theory has progressed considerably. Much of the recent progress was sparked by a remarkable paper of Minahan and Zarembo [22] which shows that the spectrum of one loop anomalous dimensions of operators dual to closed string states, in a sub sector of the theory, gives rise to an integrable SO(6) spin chain. This result can be generalized to include the full set of local operators of the theory [23]. The integrable spin chain model describing the full planar one loop spectrum of anomalous dimensions can be solved by Bethe-Ansatz techniques [23]. Clearly, it is desirable to find a similar approach for operators dual to open strings. A naive generalization is frustrated by the fact that, since the open string and giant can exchange momentum, the number of sites of the open string lattice becomes a dynamical variable.[3] This was circumvented in [25] by introducing a Cuntz oscillator chain. Restricting to the SU(2) sector, the spin chain is obtained by mapping one of the matrices, say Z, into a spin up and the other, say Y, into a spin down. In contrast to this, the Cuntz chain uses the Ys to set up a lattice which is populated by the Zs. Thus the number of sites in the Cuntz chain is fixed.

The power of the spin chain goes beyond the computation of anomalous dimensions. Indeed, the low energy description of the spin chain relevant for closed string states appearing on the field theory side matches perfectly with the low energy limit of the string action in $AdS_5 \times S^5$ [26]. This is an important result because it shows how a string action can emerge from large N gauge theory. For the open string, the coherent state expectation value of the Cuntz chain Hamiltonian reproduces the open string action for an open string attached to a sphere giant in $AdS_5 \times S^5$ [25, 17], for an open string attached to an AdS giant in $AdS_5 \times S^5$ [27] and for an open string attached to a sphere giant in a deformed $AdS_5 \times S^5$ background [28]. Recently [29], the worldsheet theory of an open string attached to a maximal giant has been studied. Evidence that the system is integrable at two loops has been obtained.

The fact that the open string can exchange momentum with the giant is reflected in the fact that there are sources and sinks (at the endpoints of the string) for the particles on the chain. The structure of these boundary interactions is complicated: since the brane can exchange momentum with the string, the brane will in general be deformed

[2] We review the definition of the restricted Schur polynomial in appendix E.

[3] An exception to this is an open string attached to a maximal giant graviton [24].

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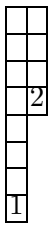
by these boundary interactions. The goal of this article is to determine this Cuntz chain Hamiltonian for multiple strings attached to an arbitrary system of giant gravitons. In particular, this entails accounting for back reaction on the giant graviton. To compute the Cuntz chain Hamiltonian, we need the two point functions of restricted Schur polynomials. It is an involved combinatoric task to compute the two point functions of restricted Schur polynomials. The required technology to compute these correlators, in the free field limit, has recently been developed in [30].[4] This was then extended to one loop, for operators dual to giants with a single string attached [32]. In this article, we extend the existing technology, allowing the one loop computation of correlators dual to giant graviton systems with an arbitrary number of strings attached. In the remainder of this introduction, we will establish notation and give a sketch of the technology we develop.

To make our discussion concrete, we mostly consider the specific example of two strings attached to a bound state of two sphere giants.[5] Note however, that most of the formulas we derive (and certainly the techniques we develop) are applicable to the general problem. Both the strings and the branes that we consider are distinguishable. In this case there are a total of six possible states. For a bound state of two sphere giant gravitons, we need to consider restricted Schur polynomials labeled by Young diagrams with two columns each with $O(N)$ boxes. Denote the number of boxes in the first column by b_0+b_1 and the number of boxes in the second column by b_0. It is natural to interpret the number of boxes in each column as the momentum of each giant. We can use the state operator correspondence (see appendices C.5 and D for further discussion) to associate a Cuntz chain state with each restricted Schur polynomial. The Cuntz chain states have six labels in total: the first two labels are b_0 and b_1 which determine the momenta of the two giants; the next two labels are the branes on which the endpoints of string one are attached and the final two labels are the branes on which the endpoints of string two are attached. We label the strings by '1' and '2'. The brane corresponding to column 1 of the Young diagram is labeled 'b' (for big brane) and the brane corresponding to column 2 of the Young diagram is labeled 'l' (for little brane). Since the second column of a Young diagram can never contain more boxes that the first column, and since the radius of the giant graviton is determined by the square root of its angular momentum, these are accurate labels. Consider a state with string 1 on big brane and string 2 on little brane. The restricted Schur polynomial (written using the graphical notation of [30, 32]) together with the corresponding Cuntz chain state are (in this case, $b_0 = 3$ and $b_1 = 4$)



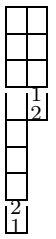
$$\longleftrightarrow |3, 4, bb, ll\rangle.$$

We will call states with strings stretching between branes "stretched string states". When

[4] For some earlier related work, see [31].

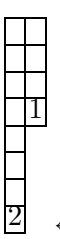
[5] In appendix F we consider a boundstate of three sphere giants with two open strings attached.

labeling the Cuntz chain state corresponding to a stretched string state, we will write the end point label corresponding to the upper index first. Thus,



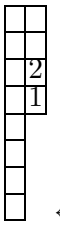
$$\longleftrightarrow |3,4,lb,bl\rangle.$$

The remaing four states are

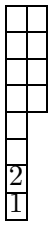


$$\longleftrightarrow |3,4,ll,bb\rangle$$

$$\longleftrightarrow |3,4,bl,lb\rangle,$$



$$\longleftrightarrow |2,6,ll,ll\rangle$$



$$\longleftrightarrow |4,2,bb,bb\rangle.$$

The construction of the operators dual to excitations described by strings stretching between the branes requires the construction of an "intertwiner" [30]. One of the results of the present article, is to provide a general construction of the intertwiner. This construction is given in appendix A. In the notation of [30], we assume that when the restricted Schur polynomial is to be reduced, string 1 is removed first and string 2 second. This implies that, when using the graphical notation, removing the box occupied by string 1 first will always leave a valid Young diagram. This choice is arbitrary, but useful for explicit computation. Once we have the form of the Hamiltonian, we can always change to a "physical basis". To obtain operators dual to giant gravitons, we take b_0 to be $O(N)$ and b_1 to be $O(1)$. We want to compute the matrix of anomalous dimensions to one loop and at large N. To compute this matrix, we need to compute the two point functions of restricted Schur polynomials. This is a hard problem: since the number of fields in the giant graviton is $O(N)$, huge combinatoric factors pile up as the coefficient of non-planar diagrams and the usual the planar approximation fails. We need to contract all of the fields in the giant gravitons exactly. The two open strings are described by the words $W^{(1)}$ and $W^{(2)}$. The six Higgs fields ϕ^i $i = 1, \ldots, 6$, of the $\mathcal{N} = 4$ super Yang-Mills theory can be grouped into the following complex combinations

$$Z = \phi^1 + i\phi^2, \qquad Y = \phi^3 + i\phi^4, \qquad X = \phi^5 + i\phi^6.$$

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The giant gravitons are built out of the Z field; the open string words out of the Z and Y fields. Thus, the open strings carry a component of angular momentum on the S^3 that the giant wraps, as well a component parallel to the giant's angular momentum. We will normalize things so that the action of $\mathcal{N}=4$ super Yang-Mills theory on $R\times S^3$ is (we consider the Lorentzian theory and have set the radius of the S^3 to 1)

$$S=\frac{N}{4\pi\lambda}\int dt\int_{S^3}\frac{d\Omega_3}{2\pi^2}\left(\frac{1}{2}(D\phi^i)(D\phi^i)+\frac{1}{4}\big([\phi^i,\phi^j]\big)^2-\frac{1}{2}\phi^i\phi^i+\dots\right), \tag{1.1}$$

With these conventions,

$$\langle Z^\dagger_{ij}(t)Z_{kl}(t)\rangle=\frac{4\pi\lambda}{N}\delta_{il}\delta_{jk}=\langle Y^\dagger_{ij}(t)Y_{kl}(t)\rangle, \tag{1.2}$$

The open string words can be labelled as

$$(W(\{n_1,n_2,\dots,n_{L-1}\}))^i_j=(YZ^{n_1}YZ^{n_2}Y\cdots YZ^{n_{L-1}}Y)^i_j, \tag{1.3}$$

where $\{n_1,n_2,\dots,n_{L-1}\}$ are the Cuntz lattice occupation numbers. The giant is built out of Zs; the first and last letters of the open string word W are not Zs. We will always use L to denote the number of Y fields in the open string word and $J=n_1+n_2+\cdots+n_{L-1}$ to denote the number of Z fields in the open string word. The number of fields in each word is $J+L\approx L$ in the case that $J\ll L$ which we will assume in this article. For the words $W^{(1)},W^{(2)}$ to be dual to open strings, we need to take $L\sim O(\sqrt{N})$. We do not know how to contract the open strings words exactly; when contracting the open string words, only the planar diagrams are summed. To suppress the non-planar contributions we take $\frac{L^2}{N}\ll 1$. To do this we consider a double scaling limit in which the first limit takes $N\to\infty$ holding $\frac{L^2}{N}$ fixed and the second limit takes the effective genus counting parameter $\frac{L^2}{N}$ to zero. Taking the limits in this way corresponds, in the dual string theory, to taking the string coupling to zero, in the string theory constructed in a fixed giant graviton background. Since the two strings are distinguishable they are represented by distinct words and hence, in the large N limit, we have

$$\langle W^{(i)}(W^{(j)})^\dagger\rangle\propto\delta^{ij}.$$

When computing a correlator of two restricted Schur polynomials, the fields belonging to the giants in the two systems of excited giant gravitons are contracted amongst each other, the fields in the first open string of each are contracted amongst each other and the fields in the second open string are contracted amongst each other. We drop the contributions coming from contractions between Zs in the open strings and Zs associated to the brane system, as well as contractions between Zs in different open string words. When computing two point functions in free field theory, if the number of boxes in the representation R is less than[6] $O(N^2)$ and the numbers of Z's in the open string is $O(1)$, the contractions between any Zs in the open string and the rest of the operator are suppressed in the large

[6]When the number of operators in the Young diagram is $O(N^2)$, the operator is dual to an LLM geometry [33].

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N limit [34]. Contractions between Zs in different open string words are non planar and are hence subleading (clearly there are no large combinatoric factors that modify this).

An important parameter of our excited giant graviton system is $N-b_0$. This parameter can scale as $O(N)$, $O(\sqrt{N})$ or $O(1)$. In section 2, we will see that when $N-b_0$ is $O(1)$ the sphere giant boundary interaction is $O(\frac{1}{N})$, when $N-b_0$ is $O(\sqrt{N})$ the boundary interaction is $O(\frac{1}{\sqrt{N}})$ and when $N-b_0$ is $O(N)$, the boundary interaction is $O(1)$. Since we want to explore the dynamics arising from the boundary interaction, we will assume that $N-b_0$ is $O(N)$.

The subspace of states reached by attaching two open strings to a giant graviton boundstate system is dynamically decoupled (from subspaces obtained by attaching a different number of open strings) at large N. It is possible to move out of this subspace by the process in which the word W "fragments" thereby allowing Ys to populate more than a single box in R. In the dual string theory this corresponds to a splitting of the original string into smaller strings, which are still attached to the giant. This process was considered in [30] and from that result we know that it does not contribute in the large N limit. One could also consider the process in which the open string detaches from the brane boundstate and is emitted as a closed string state, so that it no longer occupies any box in R. This process (decay of the excited giant boundstate by gravitational radiation) also does not contribute in the large N limit [16, 30].

Since the giant boundstate and the open string can exchange momentum, the value of J is not a parameter that we can choose, but rather, it is determined by the dynamics of the problem. Cases in which J becomes large correspond to the situation in which a lot of momentum is transferred from the giant to the open string, presumably signaling an instability. See [17] for a good physical discussion of this instability. In cases where J is large, back reaction is important and the approximations we are employing are no longer valid. This will happen when J becomes $O(\sqrt{N})$ since the assumption that we can drop non-planar contributions when contracting the open string words breaks down. Essentially this is because as more and more Zs hop onto the open string, it is starting to grow into a state which is eventually best described as a giant graviton itself. One can also no longer neglect the contractions between any Zs in the open string and the rest of the operator, presumably because the composite system no longer looks like a string plus giant (which can be separated nicely) but rather, it starts to look like one large deformed threebrane. Thus, the fact that our approximation breaks down has a straight forward interpretation: We have set up our description by assuming that the operator we study is dual to a threebrane with an open string attached. This implies that our operator can be decomposed into a "threebrane piece" and a "string piece". These two pieces are treated very differently: when contracting the threebrane piece, all contractions are summed; when contracting the string piece, only planar contractions are summed. Contractions between the two pieces are dropped. When a large number of Zs hop onto the open string our operator is simply not dual to a state that looks like a threebrane with an open string attached and our approximations are not valid. We are not claiming that this operator can not be studied using large N techniques - it may still be possible to set up a systematic $1/N$ expansion. We are claiming that the diagrams we have summed do not give this approximation.

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It is useful to decompose the potential for the scalars into D terms and F terms. The advantage of this decomposition is that it is known that at one loop, the D term contributions cancel with the gauge boson exchange and the scalar self energies [35]. Consequently we will only consider the planar interactions arising from the F term. The F term interaction preserves the number of Y's (the lattice is not dynamical) and allows impurities (the Zs) to hop between neighboring sites. The bulk interactions are described by the Hamiltonian

$$H_{\text{bulk}} = 2\lambda \sum_{l=1}^{L} \hat{a}_l^\dagger \hat{a}_l - \lambda \sum_{l=1}^{L-1} \left(\hat{a}_l^\dagger \hat{a}_{l+1} + \hat{a}_l \hat{a}_{l+1}^\dagger \right), \tag{1.4}$$

where

$$\hat{a}_i \hat{a}_i^\dagger = I, \qquad \hat{a}_i^\dagger \hat{a}_i = I - |0\rangle\langle 0|. \tag{1.5}$$

The interested reader is referred to [17] for the derivation of this result. To obtain the full Hamiltonian, we need to include the boundary interactions arising from the string/brane system interaction. This interaction introduces sources and sinks for the impurities at the boundaries of the lattice. The boundary interaction allows Zs to hop from the string onto the giant, or from the giant onto the string. Since the number of Zs gives the angular momentum of the system in the plane that the giant is orbiting in, the boundary interaction allows the string and the brane to exchange angular momentum. We can classify the different types of boundary interaction depending on whether momentum flows from the string to the brane or from the brane to the string. Consider the interaction that allows a Z to hop from the first or last site of either string onto the giant. In this process the string loses momentum to the giant graviton. We call this a "hop off" process because a Z has hopped off the string. The opposite process in which a Z hops off the brane and onto the string is called a "hop on" process. In the "hop on" process the giant loses momentum to the string. In addition to these momentum exchanging processes, there is also a boundary interaction in which a Z belonging to the giant "kisses" the first (or last) Y in the open string word so that no momentum is exchanged. We call this the kissing interaction.

In appendix C we will derive a set of identities that allow us to compute the term in the Hamiltonian describing the "hop off" process. These identities make extensive use of the technology for computing restricted characters which is developed in appendix B. We will now explain what we mean by a restricted character. Let R be an irrep of S_n and let R_1 be an irrep of S_{n-m} with $0 < m < n$. If we restrict ourselves to elements $\sigma \in S_{n-m}$, then $\Gamma_R(\sigma)$ will, in general, subduce a number of irreps of S_{n-m}. One of these irreps is R_1. A restricted character $\chi_{R,R_1}(\sigma)$ is obtained by tracing the matrix representing σ in irrep R, $\Gamma_R(\sigma)$, over the R_1 subspace. If $\sigma \in S_{n-m}$ this simply gives the character of σ in irrep R_1. Our technology allows the computation of $\chi_{R,R_1}(\sigma)$ even when $\sigma \notin S_{n-m}$, in which case $\chi_{R,R_1}(\sigma)$ does not have an obvious group theory interpretation. The basic idea we exploit in constructing the "hop off" process is simple to state: In a string-giant system, whenever a Z field hops past the borders of the open string word W, the resulting restricted Schur polynomial decomposes into a sum of two types of systems, one is a giant with a closed string and another is a string-giant system where the giant is now bigger. In the large N limit only this second type needs to be considered. Our identities express this decomposition.

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Since the Hamiltonian must be Hermitian, we can obtain the "hop on" term by daggering the "hop off" term. Finally, we obtain the momentum conserving boundary interaction by expressing the kiss as a hop on followed by a hop off. This determines the complete Cuntz oscillator chain Hamiltonian needed for a one loop computation of the anomalous dimensions of operators dual to excited giant graviton bound states. This derivation of the Cuntz chain Hamiltonian, which is the main technical result of this article, is given in section 2.

The resulting Hamiltonian clearly reflects the worldsheet structure of the open strings that are interacting. This explains how the Chan-Paton factors associated with strings in a multi-brane system dynamically emerge from Yang-Mills theory: they emerge from the symmetric group labels of the restricted Schur polynomials. Our Hamiltonian treats string 1 and string 2 differently. This is not at all surprising, since when we built our operator we treated the two strings differently. In section 3 we describe a new "physical basis" singled out by the requirement that the two strings enter on an equal footing. In section 4 we present our conclusions.

Strings stretching between giants in AdS can be realized as solutions to the Born-Infeld action describing the world volume dynamics of these branes [19]. In this work the Gauss law is enforced by the construction of consistent solutions to the equations of motion on a compact space. In the work [20] the one loop anomalous dimension of operators representing a string attached to a two brane bound state was considered. One of the branes was taken to be a maximal giant to simplify the computation. For two coincident branes, the one-loop anomalous dimension for an open string is twice the answer for a single brane. This is the first, to the best of our knowledge, hint of the dynamical emergence of Chan-Paton factors for open strings on coincident branes. The demonstration of [20] identifies the extra factor of 2 with the trace over the indices of the enhanced gauge group associated to coincident branes. Our demonstration proves this identification: we can follow the Chan-Paton indices in the tree level transitions of two open strings. Further, using the technology we develop, it is straight forward (but technically involved) to generalize this result to a bound state of m branes, where we expect a $\mathrm{U}(m)$ gauge theory to emerge. As an example, in appendix F we consider a boundstate of three sphere giants. In this case, a $\mathrm{U}(3)$ gauge theory emerges.

2. Cuntz chain hamiltonian

In this section we will derive the form of the terms in the Hamiltonian describing the string boundary interactions. This will allow us to compute the complete Cuntz chain Hamiltonian, since the bulk Hamiltonian has already been given in (1.4).

2.1 Hop off interaction

We start by deriving the hop off interaction. The F term vertex allows a Z and a Y to change position within a word. The hopping interaction corresponds to the situation in which a Z hops past the Y marking the end point of the string, i.e. a Z hops off the string

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and onto the giant. Concretely, when acting on either open string, this hop takes

$$W(\{n_1, n_2, \ldots, n_{L-1}\}) \to ZW(\{n_1 - 1, n_2, \ldots, n_{L-1}\}) \quad \text{or}$$
$$W(\{n_1, n_2, \ldots, n_{L-1}\}) \to W(\{n_1, n_2, \ldots, n_{L-1} - 1\})Z.$$

To determine the corresponding term in the interaction Hamiltonian, we need to be able to express objects like $\chi^{(2)}_{R,R''}(Z, ZW^{(1)}, W^{(2)})$ in terms of $\chi^{(2)}_{S,S''}(Z, W^{(1)}, W^{(2)})$ where S is a Young diagram with one more box than R.[7] This is easily achieved by inverting the identities derived in appendix C. To get the hop off interaction in the Hamiltonian, we rewrite the identities in terms of normalized Cuntz chain states.

+1 → 1 hop off interaction. This term in the Hamiltonian describes the hop off process in which a Z hops out of the first site of string 1. We write $+1 \to 1$ to indicate that the string before the hop has one extra Z in its first site.

$$H_{+1\to 1}\begin{bmatrix} |b_0-1, b_1, bb, ll\rangle \\ |b_0-1, b_1, ll, bb\rangle \\ |b_0-1, b_1, bl, lb\rangle \\ |b_0-1, b_1, lb, bl\rangle \\ |b_0-2, b_1+2, ll, ll\rangle \\ |b_0, b_1-2, bb, bb\rangle \end{bmatrix} = -\lambda\sqrt{1-\frac{b_0}{N}}M_1 \begin{bmatrix} |b_0-1, b_1+1, bb, ll\rangle \\ |b_0, b_1-1, ll, bb\rangle \\ |b_0-1, b_1+1, bl, lb\rangle \\ |b_0, b_1-1, lb, bl\rangle \\ |b_0-1, b_1+1, ll, ll\rangle \\ |b_0, b_1-1, bb, bb\rangle \end{bmatrix}, \tag{2.1}$$

where

$$M_1 = \begin{bmatrix} -(b_1)_1^2 & \frac{1}{b_1(b_1+1)^2} & 0 & \frac{(b_1)_0}{b_1+1} & \frac{(b_1)_1}{b_1+1} & -\frac{(b_1)_1}{b_1(b_1+1)} \\ -\frac{1}{(b_1+2)(b_1+1)^2} & -(b_1)_1^2 & -\frac{(b_1)_2}{b_1+1} & 0 & -\frac{(b_1)_1}{(b_1+1)(b_1+2)} & -\frac{(b_1)_1}{b_1+1} \\ -\frac{(b_1)_1}{(b_1+1)(b_1+2)} & \frac{(b_1)_1}{b_1+1} & -(b_1)_1(b_1)_2 & 0 & -\frac{b_1}{(b_1+1)^2} & \frac{1}{(b_1+1)^2} \\ -\frac{(b_1)_1}{b_1+1} & -\frac{(b_1)_1}{b_1(b_1+1)} & 0 & -(b_1)_0(b_1)_1 & \frac{1}{(b_1+1)^2} & \frac{b_1+2}{(b_1+1)^2} \\ -\frac{(b_1)_2}{b_1+1} & 0 & \frac{1}{b_1+2} & 0 & -(b_1)_1(b_1)_2 & 0 \\ 0 & \frac{(b_1)_0}{b_1+1} & 0 & -\frac{1}{b_1} & 0 & -(b_1)_0(b_1)_1 \end{bmatrix},$$

and

$$(b_1)_n = \frac{\sqrt{b_1+n-1}\sqrt{b_1+n+1}}{b_1+n}.$$

The term in the Hamiltonian describing the process in which the Z hops out of the last site of string 1 is described by swapping the labels of the endpoints of the open strings. Concretely, it is given by

$$H_{1+\to 1}\begin{bmatrix} |b_0-1, b_1, bb, ll\rangle \\ |b_0-1, b_1, ll, bb\rangle \\ |b_0-1, b_1, lb, bl\rangle \\ |b_0-1, b_1, bl, lb\rangle \\ |b_0-2, b_1+2, ll, ll\rangle \\ |b_0, b_1-2, bb, bb\rangle \end{bmatrix} = -\lambda\sqrt{1-\frac{b_0}{N}}M_1 \begin{bmatrix} |b_0-1, b_1+1, bb, ll\rangle \\ |b_0, b_1-1, ll, bb\rangle \\ |b_0-1, b_1+1, lb, bl\rangle \\ |b_0, b_1-1, bl, lb\rangle \\ |b_0-1, b_1+1, ll, ll\rangle \\ |b_0, b_1-1, bb, bb\rangle \end{bmatrix}, \tag{2.2}$$

where M_1 is the matrix given above. We write $1+ \to 1$ to indicate that the string before the hop has one extra Z in its last site.

[7] The number of primes on the label of the restricted Schur polynomial indicates how many boxes are dropped, i.e. R'' is obtained by dropping two boxes from R.

+2 → 2 hop off interaction. This term in the Hamiltonian describes the hop off process in which a Z hops out of the first site of string 2.

$$H_{+2\to 2}\begin{bmatrix}|b_0-2,b_1+1,bb,ll\rangle\\ |b_0-1,b_1-1,ll,bb\rangle\\ |b_0-2,b_1+1,bl,lb\rangle\\ |b_0-1,b_1-1,lb,bl\rangle\\ |b_0-2,b_1+1,ll,ll\rangle\\ |b_0-1,b_1-1,bb,bb\rangle\end{bmatrix}=-\lambda\sqrt{1-\frac{b_0}{N}}M_2\begin{bmatrix}|b_0-1,b_1,bb,ll\rangle\\ |b_0-1,b_1,ll,bb\rangle\\ |b_0-1,b_1,bl,lb\rangle\\ |b_0-1,b_1,lb,bl\rangle\\ |b_0-1,b_1,ll,ll\rangle\\ |b_0-1,b_1,bb,bb\rangle\end{bmatrix}, \tag{2.3}$$

where

$$M_2=\begin{bmatrix} -(b_1)_1^2 & -\frac{1}{(b_1+2)(b_1+1)^2} & -\frac{(b_1)_1}{(b_1+1)(b_1+2)} & -\frac{(b_1)_1}{b_1+1} & 0 & -\frac{(b_1)_2}{b_1+1}\\ \frac{1}{b_1(b_1+1)^2} & -(b_1)_1^2 & \frac{(b_1)_1}{b_1+1} & -\frac{(b_1)_1}{(b_1+1)b_1} & \frac{(b_1)_0}{b_1+1} & 0\\ 0 & -\frac{(b_1)_2}{b_1+1} & -(b_1)_1(b_1)_2 & 0 & 0 & \frac{1}{b_1+2}\\ \frac{(b_1)_0}{b_1+1} & 0 & 0 & -(b_1)_0(b_1)_1 & -\frac{1}{b_1} & 0\\ -\frac{(b_1)_1}{b_1(b_1+1)} & -\frac{(b_1)_1}{b_1+1} & \frac{1}{(b_1+1)^2} & \frac{b_1+2}{(b_1+1)^2} & -(b_1)_1(b_1)_0 & 0\\ \frac{(b_1)_1}{b_1+1} & -\frac{(b_1)_1}{(b_1+1)(b_1+2)} & -\frac{b_1}{(b_1+1)^2} & \frac{1}{(b_1+1)^2} & 0 & -(b_1)_2(b_1)_1 \end{bmatrix}.$$

Notice that these interactions (as is the case for all of the boundary interactions) are highly suppressed for a maximal giant [24]. The term in the Hamiltonian describing the process in which the Z hops out of the last site of string 2 is described by swapping the labels of the endpoints of the open strings.

The function $(b_1)_n$ also appears in the Hamiltonian relevant for a single string attached to a giant [32]. Notice that $(b_1)_n$ vanishes when $b_1 = 1-n$, but tends to 1 very rapidly as b_1 is increased from this value. The diagonal terms in the Hamiltonian with a $(b_1)_1$ factor will thus vanish when $b_1 = 0$. The radius of each giant is determined by their momentum. Since b_1 is the difference in momentum of the two giants, $b_1 = 0$ corresponds to coincident giants. Thus, $(b_1)_n$ is switching off short distance interactions. The hop off Hamiltonian does not generate illegal Young diagrams from legal ones precisely because these interactions are switched off.

It may seem puzzling that the boundary interaction has the universal strength $\sqrt{1-\frac{b_0}{N}}$ regardless of which end the Z peels off. Indeed, any local boundary interaction should only know about the boundary which is participating. Since the string end points are on branes of different sizes, one would expect two different strengths for the two endpoints. This universal strength is an artifact of the limit we consider. We take b_0 to be $O(N)$ and b_1 to be $O(1)$. The strength $\sqrt{1-\frac{b_0}{N}}$ arises from the normalization of the Cuntz oscillator states for the string endpoint attached to the smaller giant graviton (see appendix D for these normalizations). Similarly, the strength $\sqrt{1-\frac{b_0+b_1}{N}}$ arises from the normalization of the Cuntz oscillator states for the string endpoint attached to the larger giant graviton. In the limit we consider

$$\sqrt{1-\frac{b_0+b_1}{N}}-\sqrt{1-\frac{b_0}{N}}=O\Big(\frac{1}{N}\Big).$$

Physically, taking $b_0 = O(N)$ and $b_1 = O(1)$ implies that the two branes are very nearly coincident.

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Finally, note that the structure of the hop on and hop off interactions, clearly reflect the fact that the open strings attached to the giants are orientable.

2.2 Hop on interaction

Since $\mathcal{N} = 4$ super Yang-Mills theory is a unitary conformal field theory, we know that the spectrum of anomalous dimensions of the theory is real. This implies that the energy spectrum of our Cuntz chain Hamiltonian must be real and hence the Hamiltonian must be Hermitian. Thus, the hop on term in the Hamiltonian is given by the Hermitian conjugate of the hop off term.

To give an example, we will now derive the term in the Hamiltonian describing the process in which a Z from the brane hops into the first site of string 1. Let $|\psi\rangle$ denote the state with a brane of momentum $P_{\text{brane}} = P$ and a string of momentum $P_{\text{string}} = p$ and $|\phi\rangle$ denote the state with $P_{\text{brane}} = P + 1$ and $P_{\text{string}} = p - 1$. Then,

$$H_{+1\to 1}|\psi\rangle = -\lambda\sqrt{1-\frac{b_0}{N}}M_1|\phi\rangle, \tag{2.4}$$

and

$$\langle\phi'|H_{+1\to 1}|\psi\rangle = -\lambda\sqrt{1-\frac{b_0}{N}}\langle\phi'|M_1|\phi\rangle = -\lambda\sqrt{1-\frac{b_0}{N}}(M_1)_{\phi'\phi}.$$

Daggering we find (keep in mind that M_1 is real)

$$\begin{aligned}\langle\psi|H_{1\to+1}|\phi'\rangle &= (\langle\phi'|H_{+1\to 1}|\psi\rangle)^\dagger \\ &= -\lambda\sqrt{1-\frac{b_0}{N}}\langle\phi|(M_1)^T|\phi'\rangle \\ &= -\lambda\sqrt{1-\frac{b_0}{N}}\left((M_1)^T\right)_{\phi\phi'}.\end{aligned}$$

Thus we obtain

$$H_{1\to+1}|\phi\rangle = -\lambda\sqrt{1-\frac{b_0}{N}}N_1|\psi\rangle, \tag{2.5}$$

with $N_1 = (M_1)^T$.

2.3 Kissing interaction

The kissing interaction corresponds to the Feynman diagram shown on the left in figure 1. Notice that the number of Z fields in the giant is unchanged by this process so that the string and brane do not exchange momentum by this process. As far as the combinatorics goes, we can model the kissing interaction as a hop on (the string) followed by a hop off. We know both the hop on and hop off terms so the kissing interaction follows. This is illustrated by the Feynman diagram shown on the right in figure 1. The kissing interaction must be included for both endpoints of both strings.

A straight forward computation easily gives

$$H_{\text{kissing}} = \lambda\left(1-\frac{b_0}{N}\right)\mathbf{1}, \tag{2.6}$$

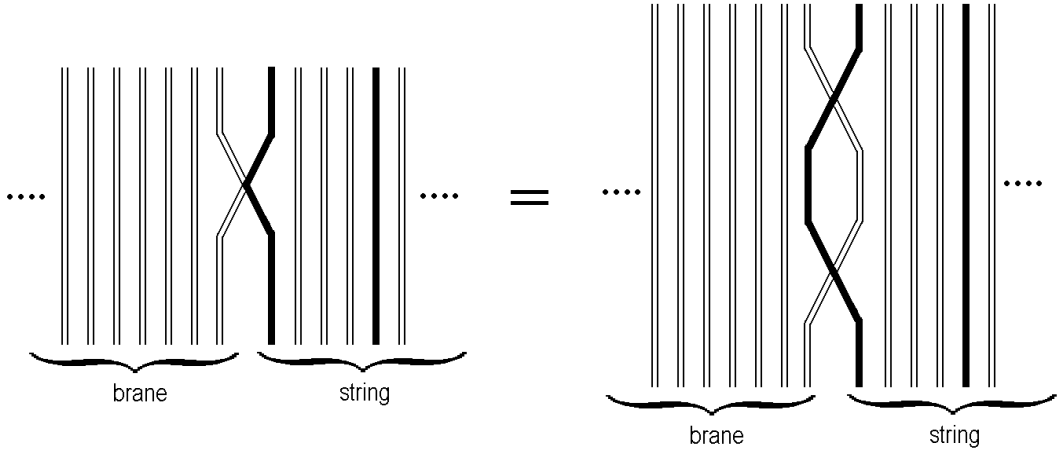

Figure 1: The Feynman diagram on the left shows the kissing interaction. The white ribbons are Z fields, the black ribbons are Y fields. The interacting black ribbon shown marks the beginning of the string; there are 3 Zs in the first site of the string. The Feynman diagram on the right shows a hop on interaction followed by a hop off interaction. If you shrink the composite hop on/hop off interaction to a point, you recover the kissing interaction.

for each endpoint of either string. In this formula **1** is the identity operator.

The fact that the kissing interaction comes out proportional to the identity operator is a non-trivial check of our hop on and hop off interactions. Indeed, the contraction of the F term vertex which leads to the kissing interaction removes an adjacent Z and Y and then replaces them in the same order. Thus, the kissing interaction had to come out proportional to the identity. The careful reader may worry that this is not in fact true - indeed, the restricted Schur polynomial includes terms in which the open string word is traced and terms in which the two open string words are multiplied. For these terms there is no Z next to the word to "do the kissing". Precisely these terms were considered in appendix C.5. They do not contribute at large N.

2.4 Validity of the Cuntz chain hamiltonian

We have made a number of approximations. When contracting the open string words, only the planar diagrams have been summed. The non-planar contributions can only be neglected if $\frac{L^2}{N} \ll 1$. Contributions coming from contractions between Zs in the open strings and Zs associated to the brane system have been dropped. When computing two point functions in free field theory, if the number of boxes in the representation R is less than $O(N^2)$ and the numbers of Z's in the open string is $O(1)$, the contractions between any Zs in the open string and the rest of the operator are suppressed in the large N limit [34]. Contractions between Zs in different open string words have been dropped because they are non planar and are hence subleading. No large combinatoric factors modify this. Finally, when J is large, back reaction is important and the approximations we are employing are no longer valid. When J becomes $O(\sqrt{N})$ the assumption that we can drop non-planar contributions when contracting the open string words breaks down.

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3. Interpretation

The operators we are studying are dual to giant gravitons with open strings attached. Since the giant gravitons have finite volume, the Gauss Law implies that the total charge on each giant must vanish - there must be the same number of strings leaving each brane as there are arriving on each brane. These operators do indeed satisfy these non-trivial constraints [20], providing convincing evidence for the proposed duality. The low energy dynamics of the open strings attached to the giant gravitons is a Yang-Mills theory. This new *emergent* $3+1$ dimensional Yang-Mills theory is not described as a local field theory on the S^3 on which the original Yang-Mills theory is defined - it is local on a new space, the world volume of the giant gravitons [20, 36]. This new space emerges from the matrix degrees of freedom participating in the Yang-Mills theory. Reconstructing this emergent gauge theory may provide a simpler toy model that will give us important clues into reconstructing the full $AdS_5 \times S^5$ quantum gravity. In this section, our goal is to make contact with this emergent Yang-Mills dynamics.

3.1 Dynamical emergence of Chan-Paton factors

Return to the $H_{+1\to 1}$ hop off interaction obtained in section 2.1. Recall that this corresponds to the interaction in which a Z hops out of the first site of string 1. If we expand the matrix M_1 for large b_1, we find

$$M_1 = \sum_{n=0}^{\infty} M_1(n) b_1^{-n}. \tag{3.1}$$

The leading order $M_1(0)$ is simply $-\mathbf{1}$ with $\mathbf{1}$ the 6×6 identity matrix. The Z simply hops off the string and onto the brane without much rearranging of the system. This is the dominant process. Next, consider the term of order b_1^{-1}. It is simple to compute

$$M_1(1) = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \end{bmatrix}. \tag{3.2}$$

The radius of the giant graviton R_g is related to its momentum P by $R_g = \sqrt{\frac{P}{N}}$. The giant orbits with a radius $R = \sqrt{1 - R_g^2}$. For the two giants in the bound state we are considering we have $P_1 = b_0$ and $P_2 = b_0 + b_1$. Using the fact that $b_0 = O(N)$ and $b_1 = O(1)$ it is simple to verify that both the difference in the radii of the two giants and the difference in the radii of their orbits is proportional to b_1. Thus, a b_1^{-1} dependence indicates a potential with an inverse distance dependence which is the correct dependence for massless particles moving in $3+1$ dimensions. In figure 2 we have represented the transitions implied by $M_1(1)$ graphically. Transitions between any two adjacent Young diagrams are allowed.

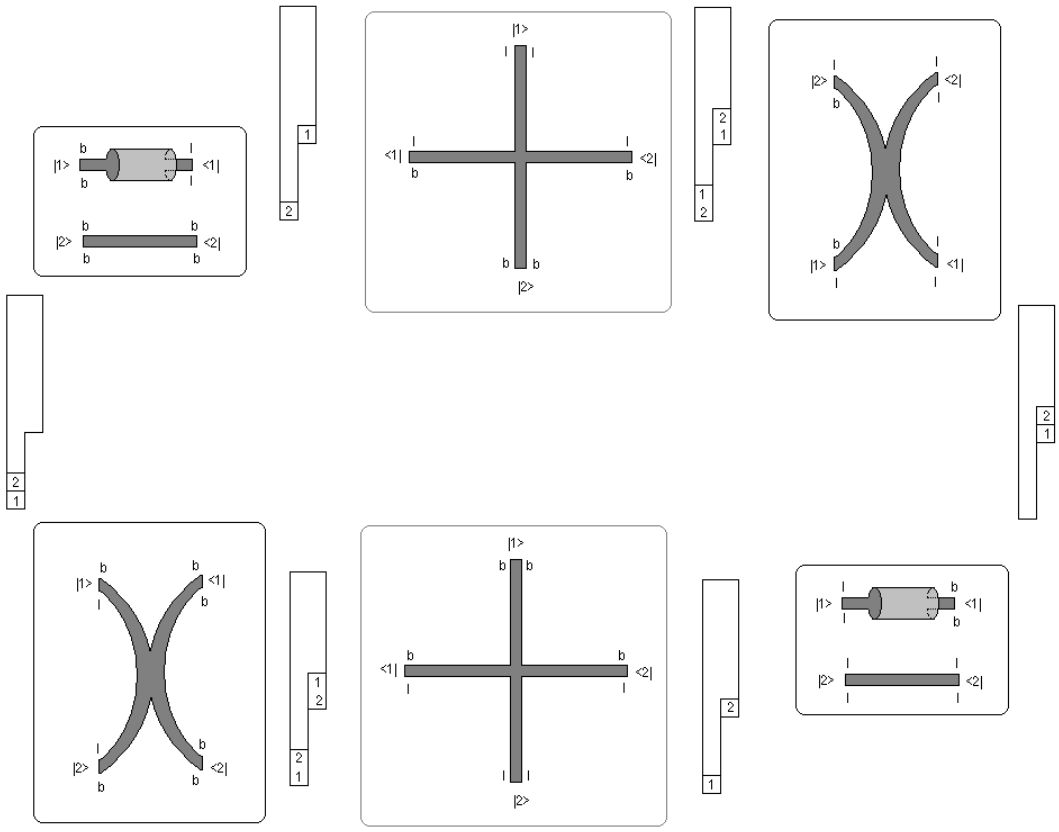
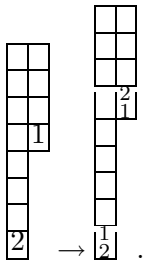


Figure 2: The order b_1^{-1} terms in the hop off interaction. This interaction allows a transition between the operators described by any two adjacent Young diagrams. The figures between the Young diagram show the open string diagram relevant for the clockwise transition. The kets are associated to the open string states before the transition; the bras to the states after the transition. The end point labels 'b' and 'l' are for big brane and little brane.

As an example, consider the transition



The upper label of string 1 has moved. In all of the transitions shown, the upper index of string 1 always moves, so that it is natural to associate the upper index of string 1 with the first site of string one, and to look for an interpretation of this interaction in terms of open string processes that involve the upper index of string 1. The figures between the Young diagram show that there is indeed a natural interpretation for these transitions. *It is clear that our Cuntz oscillator dynamics illustrates how the Chan-Paton factors for open strings*

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propagating on multiple branes arise dynamically. Drawing all possible ribbon diagrams correctly accounts for both $M_1(0)$ and $M_1(1)$.

3.2 Physical basis

Although the interpretation of the b_1^{-1} terms is encouraging, there are extra higher order corrections ($M_1(2)b_1^{-2}$, $M_1(3)b_1^{-3}$ and higher orders) that do not seem to have a natural open string interpretation. In addition to this, the interaction we have obtained depends on the open string words describing each open string, the Young diagram describing the brane bound state system as well as the order in which the strings were attached. This dependence on the order in which the strings are attached is not physically sensible.

It is natural to expect that the resolution to these two puzzles is connected. Recall that when constructing the restricted Schur polynomial we have assumed that when computing reductions, string 1 is removed first and string 2 second. This arbitrary choice defines a basis for the Cuntz oscillator chain. We interpret the unphysical features of our interactions, described in the previous paragraph, as reflecting a property of the basis it is written in and not as an inherent problem with the interaction. In this section we will define a new physical basis, singled out by the requirement that the boundary interaction does not depend on the order in which the open strings are attached.

A few comments are in order. A basis for the $\frac{1}{2}$ BPS states (giants with no open strings attached) is provided by the taking traces of Z or by taking subdeterminants or by the Schur polynomials. These are three perfectly acceptable bases, since using any single one of these bases we can generate, by taking linear combinations of the elements of the basis considered, a member from every $\frac{1}{2}$ BPS multiplet [14]. From a physical point of view, these different bases are not on an equal footing: the Schur polynomial is the most useful. Indeed, the Schur polynomials diagonalize the matrix of two point correlators (Zamolodchikov metric) so that they can be put into correspondence with the (orthogonal) states of a Fock space. In the same way, the basis for excited giants gravitons we have been considering is a perfectly acceptable basis. However, it is the operators in the physical basis (defined below) that have a good physical interpretation.

Denote our two strings by string A and string B. The state obtained by attaching string A first will be denoted by $|b_0, b_1, x_A y_A, x_B y_B\rangle$, where $x_A y_A$ are the endpoints of string A and $x_B y_B$ are the endpoints of string B. The state obtained by attaching string B first will be denoted by $|b_0, b_1, x_B y_B, x_A y_A\rangle\rangle$. In each subspace of sharp giant graviton momentum (definite b_0 and b_1), we can write the following relation between these two sets of states

$$\begin{bmatrix} |b_0, b_1, bb, ll\rangle \\ |b_0, b_1, ll, bb\rangle \\ |b_0, b_1, bl, lb\rangle \\ |b_0, b_1, lb, bl\rangle \\ |b_0, b_1, ll, ll\rangle \\ |b_0, b_1, bb, bb\rangle \end{bmatrix} = PT \begin{bmatrix} |b_0, b_1, bb, ll\rangle\rangle \\ |b_0, b_1, ll, bb\rangle\rangle \\ |b_0, b_1, bl, lb\rangle\rangle \\ |b_0, b_1, lb, bl\rangle\rangle \\ |b_0, b_1, ll, ll\rangle\rangle \\ |b_0, b_1, bb, bb\rangle\rangle \end{bmatrix}, \tag{3.3}$$

where

$$P = \begin{bmatrix} 0&1&0&0&0&0 \\ 1&0&0&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \end{bmatrix} \qquad \text{and}$$

$$T = \begin{bmatrix} \left(1-\frac{1}{(b_1+1)^2}\right) & \frac{1}{(b_1+1)^2} & -\frac{1}{(b_1+1)}\sqrt{1-\frac{1}{(b_1+1)^2}} & -\frac{1}{(b_1+1)}\sqrt{1-\frac{1}{(b_1+1)^2}} & 0 & 0 \\ \frac{1}{(b_1+1)^2} & \left(1-\frac{1}{(b_1+1)^2}\right) & \frac{1}{(b_1+1)}\sqrt{1-\frac{1}{(b_1+1)^2}} & \frac{1}{(b_1+1)}\sqrt{1-\frac{1}{(b_1+1)^2}} & 0 & 0 \\ \frac{1}{(b_1+1)}\sqrt{1-\frac{1}{(b_1+1)^2}} & -\frac{1}{(b_1+1)}\sqrt{1-\frac{1}{(b_1+1)^2}} & \left(1-\frac{1}{(b_1+1)^2}\right) & -\frac{1}{(b_1+1)^2} & 0 & 0 \\ \frac{1}{(b_1+1)}\sqrt{1-\frac{1}{(b_1+1)^2}} & -\frac{1}{(b_1+1)}\sqrt{1-\frac{1}{(b_1+1)^2}} & -\frac{1}{(b_1+1)^2} & \left(1-\frac{1}{(b_1+1)^2}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The matrix T is determined by the subgroup swap rule of [30]. It is satisfying that $PT \times PT = 1$. It is straight forward to check that

$$H_{+1\to 1} = A_{2\to 1}\, PT\, H_{+2\to 2}\, A_{1\to 2}\, PT, \tag{3.4}$$

where

$$\begin{bmatrix} |b_0-2, b_1+2, bb, ll\rangle \\ |b_0-1, b_1, ll, bb\rangle \\ |b_0-2, b_1+2, bl, lb\rangle \\ |b_0-1, b_1, lb, bl\rangle \\ |b_0-2, b_1+2, ll, ll\rangle \\ |b_0-1, b_1, bb, bb\rangle \end{bmatrix} = A_{2\to 1} \begin{bmatrix} |b_0-1, b_1, bb, ll\rangle \\ |b_0-1, b_1, ll, bb\rangle \\ |b_0-1, b_1, bl, lb\rangle \\ |b_0-1, b_1, lb, bl\rangle \\ |b_0-1, b_1, ll, ll\rangle \\ |b_0-1, b_1, bb, bb\rangle \end{bmatrix}, \qquad \text{and}$$

$$\begin{bmatrix} |b_0-2, b_1+1, bb, ll\rangle \\ |b_0-1, b_1-1, ll, bb\rangle \\ |b_0-2, b_1+1, bl, lb\rangle \\ |b_0-1, b_1-1, lb, bl\rangle \\ |b_0-2, b_1+1, ll, ll\rangle \\ |b_0-1, b_1-1, bb, bb\rangle \end{bmatrix} = A_{1\to 2} \begin{bmatrix} |b_0-2, b_1+1, bb, ll\rangle \\ |b_0-2, b_1+1, ll, bb\rangle \\ |b_0-2, b_1+1, bl, lb\rangle \\ |b_0-2, b_1+1, lb, bl\rangle \\ |b_0-3, b_1+3, ll, ll\rangle \\ |b_0-1, b_1-1, bb, bb\rangle \end{bmatrix}.$$

Denote the similarity transformation which takes us to the physical basis by S. In this basis, we denote $H_{+1\to 1}$ by $\hat{H}_{+1\to 1}$ and $H_{+2\to 2}$ by $\hat{H}_{+2\to 2}$. Clearly

$$\hat{H}_{+1\to 1} = S H_{+1\to 1} S^{-1}, \qquad \hat{H}_{+2\to 2} = S H_{+2\to 2} S^{-1}.$$

The transformation S is now determined by the requirement

$$\hat{H}_{+1\to 1} = P \hat{H}_{+2\to 2} P. \tag{3.5}$$

We have not yet been able to solve this equation for S. Due to the presence of $A_{1\to 2}$ and $A_{2\to 1}$ in the relation between $H_{+1\to 1}$ and $H_{+2\to 2}$, it seems that S must mix subspaces of

different giant momenta (b_0, b_1). In this case the physical basis will not have sharp giant momentum and hence the resulting states will not have a definite radius. This is not too surprising: the open strings will pull "dimples" out of the giant graviton's world volume so that the giant with an open string attached does not have a definite radius. We leave the interesting question of determining the transformation S for the future.

4. Discussion

A bound state of giant gravitons can be excited by attaching open strings. The problem of computing the anomalous dimensions of these operators can be replaced with the problem of diagonalizing a Cuntz oscillator Hamiltonian. In this article we have developed the technology needed to construct this Cuntz oscillator Hamiltonian to one loop. Firstly, we have given an algorithmic construction of the operators dual to excitations described by open strings which stretch between the branes. This involved giving an explicit construction of the intertwiner which is used to construct the relevant restricted Schur polynomial. Secondly, we have developed methods that allow an efficient evaluation of any restricted character. Our method expresses the restricted character graphically as a sum of strand diagrams. Finally, we have explained how to derive the boundary interaction terms from identities satisfied by the restricted Schur polynomials. Since the excited giant graviton operators are small excitations of BPS states, we expect that our results can be extrapolated to strong coupling and hence can be compared with results from the dual string theory. The form of our Cuntz oscillator Hamiltonian provides evidence that the excitations of the giant gravitons have the detailed interactions of an emergent gauge theory. In particular, we have demonstrated the dynamical emergence of the Chan-Paton factors of the open strings. We have also started to clarify the dictionary relating the states of the Cuntz oscillator chain to the states of string field theory on D-branes in $AdS_5 \times S^5$. Although we have mainly considered a bound state of two sphere giants with two open strings attached, our methods are applicable to an arbitrary bound state of giant gravitons with any number of open strings attached.

Our result is a generalization of the spin chains considered so far in the literature: usually the spin chain gives a description of closed strings. Our Cuntz oscillator describes the dynamics of an open string interacting with a giant graviton. Both the state of the string (described by the Cuntz chain occupation numbers) and the state of the giant graviton (the shape of the Young diagram) are dynamical in our approach.

It is worth emphasizing that the new emergent gauge symmetry is distinct from the original gauge symmetry of the theory [20]. The excited giant graviton operators [20] are obtained by taking a trace over the indices of the symmetric group matrix $\Gamma_R(\sigma)$ appearing in the sum

$$\frac{1}{(n-k)!}\sum_{\sigma\in S_n}\Gamma_R(\sigma)\mathrm{Tr}\,(\sigma Z^{\otimes n-k}W^{(1)}\cdots W^{(k)}), \qquad \text{where}$$

$$\mathrm{Tr}\,(\sigma Z^{\otimes n-k}W^{(1)}\cdots W^{(k)}) = Z^{i_1}_{i_{\sigma(1)}}Z^{i_2}_{i_{\sigma(2)}}\cdots Z^{i_{n-k}}_{i_{\sigma(n-k)}}(W^{(1)})^{i_{n-k+1}}_{i_{\sigma(n-k+1)}}\cdots(W^{(k)})^{i_n}_{i_{\sigma(n)}}.$$

The color indices of the original super Yang-Mills theory are all traced: every term in the above sum is a color singlet with respect to the gauge symmetry of the original Yang-Mills

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theory. The color indices of the new gauge theory arise from the labeling of the partial trace over $\Gamma_R(\sigma)$. In some sense we are "substituting" symmetric group indices for the original gauge theory indices. We call this mechanism "*color substitution*".

There are a number of directions in which this work can be extended. For Young diagrams with m columns we expect an emergent Yang-Mills theory with gauge group $\mathrm{U}(m)$. It would be nice to repeat the calculations we performed here in that setting.[8] Another interesting calculation would involve studying the dynamics of two giant gravitons with strings stretched between them. In general, the boundary terms will certainly have different values at each boundary (as anticipated in [17]) in which case there will be a net flow of Zs from one brane to the other. This flow of Z's will produce a force between the two giants, conjectured to be an attractive force in [17].

A very concrete application of our methods is the construction of the gauge theory operator dual to the fat magnon [37].[9] The fat magnon is a bound state of a giant graviton and giant magnons (fundamental strings). Essentially, due to the background five form flux, the giant magnon becomes fat by the Myers effect [38]. The fat magnon has the same anomalous dimension as the giant magnon. It would be nice to explicitely recover this anomalous dimension using our technology.[10]

Finally, there is now a proposal for gauge theory operators dual to brane-anti-brane states [39]. This proposal was made, at the level of the free field theory, by identifying the operators that diagonalize the two point functions of operators built from Z and $Z^\dagger$. Since these states are non-supersymmetric, corrections when the coupling is turned on are expected to be important for the physics. It would be interesting to extend the technology developed in this article to this non-supersymmetric setting.

Acknowledgments

We would like to thank Rajsekhar Bhattacharyya, Norman Ives, Sanjaye Ramgoolam, João Rodrigues, Shahin Sheikh-Jabbari and Alex Welte for pleasant discussions and/or helpful correspondence. This work is based upon research supported by the South African Research Chairs Initiative of the Department of Science and Technology and National Research Foundation. Any opinion, findings and conclusions or recommendations expressed in this material are those of the authors and therefore the NRF and DST do not accept any liability with regard thereto. This work is also supported by NRF grant number Gun 2047219.

Navigating the appendices. In the appendices we will freely make use of results obtained in the previous two articles in this series [30, 32]. A reader wishing to master the details of our analysis will need to review this background. We will now explain which results are used when. For a discussion of intertwiners see sections 2.2 and section C.1 of [30]. In appendix B we make frequent use of the subgroup swap rule which is derived in appendix D of [30]. This is perhaps the most technical result from [30, 32] that is used

[8] For the $m = 3$ case, see appendix F.

[9] The fat magnon in the plane wave background is the hedgehog of [19]

[10] We would like to thank Shahin Sheikh-Jabbari for suggesting this to us.

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in this article. For this reason, we have reviewed a concrete example in the first section of appendix B. We also use the character identity given at the end of section D.1 of [30]. The strategy for deriving the hopping identities of appendix C was given in [32].

A. Intertwiners

Intertwiners are used to construct operators dual to states with open strings stretching between giant gravitons. In this appendix we provide a general discussion of intertwiners and their construction.

A.1 Strings stretching between two branes

The Gauss Law is a strict constraint on the allowed excited brane configurations [20]: since the branes we consider have a compact world volume, the total charge on any given brane must vanish. This implies that to construct a state with strings stretching between two branes, we need at least two strings in the brane plus string system. Thus, in constructing the restricted Schur polynomial, we will need to remove at least two boxes. For concreteness, consider the case of two sphere giants, so that our restricted Schur polynomial is built with the Young diagram R that has two columns and each column has $O(N)$ boxes. R has a total of $n = O(N)$ boxes. Denote the two boxes to be removed in constructing the restricted Schur polynomial[11] by box 1 and box 2. To attach strings stretching between these two giants, the two boxes must belong to different columns. Assume that box 1 belongs to column 1 and box 2 to column 2. After restricting S_n to an S_{n-1} subgroup, representation R subduces irrep R' (whose Young diagram is obtained by removing box 1 from R) and irrep S' (whose Young diagram is obtained by removing box 2 from R). If we now further restrict to an S_{n-2} subgroup, one of the irreps subduced by R' is R'' (whose Young diagram is obtained by removing box 2 from R') and one of the irreps subduced by S' is S'' (whose Young diagram is obtained by removing box 1 from S'). Note that R'' and S'' have the same Young diagram (and hence the same dimension) but act on distinct states in the carrier space of R. The two possible intertwiners we can define map between the states belonging to R'' and the states belonging to S''.

The precise form of the intertwiners depends on the basis used for the S_{n-2} irreps $\Gamma_{R''}(\sigma)$ and $\Gamma_{S''}(\sigma)$. In writing down the intertwiner, we assume that $\Gamma_{R''}(\sigma)$ and $\Gamma_{S''}(\sigma)$ represent σ with the same matrix. With this assumption, it is possible to put the elements of the basis of the carrier space of R'' into one to one correspondence with the elements of the basis of the carrier space of S'': $|i, R''\rangle \leftrightarrow |i, S''\rangle$. We will use this correspondence below. In a suitable basis, we have

$$\Gamma_R(\sigma) = \begin{bmatrix} \Gamma_{R''}(\sigma) & 0 & \cdots \\ 0 & \Gamma_{S''}(\sigma) & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix},$$

[11] See appendix E for a quick review of restricted Schur polynomials and [20, 30, 32] for a detailed discussion.

for $\sigma \in S_{n-2}$. In constructing the restricted Schur polynomial, we also consider more general $\sigma \in S_n$. In this case, if $\sigma \notin S_{n-2}$, $\Gamma_R(\sigma)$ will not be block diagonal. Even in this more general case, we will use the labels of the S_{n-2} subduced subspaces to label the carrier space of irrep R. Denote the projection operator that projects from the carrier space of R to the R'' subspace by $P_{R\to R'\to R''}$, and the projection operator that projects from the carrier space of R to the S'' subspace by $P_{R\to S'\to S''}$. Clearly, the intertwiner which maps from S'' to R'' must take the form

$$I_{R'',S''} = P_{R\to R'\to R''} O P_{R\to S'\to S''} = \begin{bmatrix} 0 & M & \cdots \\ 0 & 0 & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}. \tag{A.1}$$

The second possible intertwiner that we can construct is given by

$$I_{S'',R''} = P_{R\to S'\to S''} O P_{R\to R'\to R''} = \begin{bmatrix} 0 & 0 & \cdots \\ M & 0 & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}.$$

We want to find a unique specification for O so that M is simply the identity matrix. For $\sigma \in S_{n-2}$ we have

$$\Gamma_R(\sigma) I_{R'',S''} = \begin{bmatrix} 0 & \Gamma_{R''}(\sigma) M & \cdots \\ 0 & 0 & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}$$

and

$$I_{R'',S''} \Gamma_R(\sigma) = \begin{bmatrix} 0 & M\Gamma_{S''}(\sigma) & \cdots \\ 0 & 0 & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}.$$

Now, by assumption, $\Gamma_{R''}(\sigma) = \Gamma_{S''}(\sigma)$ since we have $\sigma \in S_{n-2}$. Thus,

$$\big[\Gamma_R(\sigma), I_{R'',S''}\big] = \begin{bmatrix} 0 & \big[\Gamma_{R''}(\sigma), M\big] & \cdots \\ 0 & 0 & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}. \tag{A.2}$$

Applying Schur's Lemma (for irrep R'') to the right hand side implies that M is the identity matrix if and only if $\big[\Gamma_R(\sigma), I_{R'',S''}\big] = 0$ for all $\sigma \in S_{n-2}$. Clearly, for $\sigma \in S_{n-2}$ we have $\big[\Gamma_R(\sigma), P_{R\to R'\to R''}\big] = \big[\Gamma_R(\sigma), P_{R\to S'\to S''}\big] = 0$ so that

$$0 = \big[\Gamma_R(\sigma), I_{R'',S''}\big] = P_{R\to R'\to R''} \big[\Gamma_R(\sigma), O\big] P_{R\to S'\to S''}.$$

Thus, we will require

$$\big[\Gamma_R(\sigma), O\big] = 0, \qquad \forall \sigma \in S_{n-2}. \tag{A.3}$$

If we specify a condition that determines the normalization of the intertwiner, then this normalization condition and (A.3) provide the specification for O that we were looking for. The normalization of the intertwiner is fixed by demanding that

$$\mathrm{Tr}\,(M) = \dim_{R''},$$

with $\dim_{R''}$ the dimension of irrep R''. This provides a unique definition of the intertwiner.

For the example we are considering here, imagine that the S_{n-1} subgroup is obtained as

$$\mathcal{G} = \{\sigma \in S_n | \sigma(n) = n\},$$

and further that the S_{n-2} subgroup is obtained as

$$\mathcal{H} = \{\sigma \in \mathcal{G} | \sigma(n-1) = n-1\}.$$

Then the intertwiner is given by

$$I_{R'',S''} = \mathcal{N} P_{R\to R'\to R''} \Gamma_R(n, n-1) P_{R\to S'\to S''},$$

with

$$\mathcal{N}^{-1} = \frac{\mathrm{Tr}_{R'',S''}(\Gamma_R(n,n-1))}{\dim_{R''}} \equiv \sum_{i=1}^{\dim_{R''}} \frac{\langle R'', i|\Gamma_R(n,n-1)|S'', i\rangle}{\dim_{R''}}.$$

This last equation makes use of the correspondence between the bases of the carrier spaces R'' and S''. Using the technology developed in the next appendix, we find

$$\frac{\mathrm{Tr}_{R'',S''}(\Gamma_R(n,n-1))}{\dim_{R''}} = \sqrt{1 - \frac{1}{(c_1 - c_2)^2}},$$

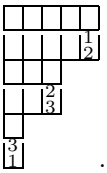
where c_1 and c_2 are the weights associated with box 1 and box 2 respectively. Note that the above trace is invariant under simultaneous similarity transformations of R'' and S''. It will however, change under general similarity transformations so that this last result is dependent on our choice of basis.

A.2 The general construction

In the previous section we have developed our discussion of the intertwiner using a system of two branes with strings stretching between them. Our conclusion however, is completely general. For any system of branes with strings stretching between the branes, the intertwiner is always given, up to normalization, by the product (projection operator)×(group element)×(projection operator). The Gauss Law forces the net charge on any given brane's worldvolume to vanish. This implies that for every string leaving a brane's worldvolume, there will be a string ending on the worldvolume. Thus, starting with any particular brane with a stretched string attached, we can follow the string to the next brane, switch to the stretched string leaving that brane, follow it and so on, until we again reach the first brane. If we move along k stretched strings before returning to the starting point, the group element is $\Gamma_R(n, n-k+1)$. The normalization factor easily follows using the results of appendix B.

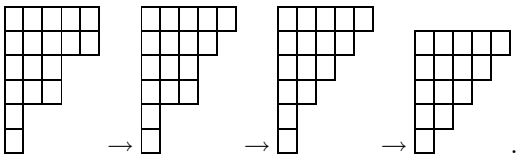
A.3 Example

Consider the excited brane system described by the diagram (see appendix E for a summary of our graphical notation)

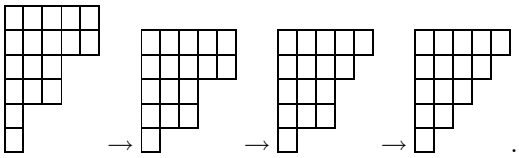


.

The boxes are labeled by the upper index in each box and the weight of box i is denoted c_i. The projector $P_{R\to R_1'''}$ projects through the following sequence of irreps



The projector $P_{R\to R_2'''}$ projects through the following sequence of irreps



The intertwiner is now given by

$$I_{12} = \mathcal{N} P_{R\to R_2'''} \Gamma_R\left((n, n-2)\right) P_{R\to R_1'''},$$

where

$$\mathcal{N}^{-1} = \frac{\mathrm{Tr}_{R_2''',R_1'''}\left(\Gamma_R\left((n,n-2)\right)\right)}{\dim_{R_1'''}} = \frac{1}{c_2-c_3}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\sqrt{1-\frac{1}{(c_1-c_3)^2}},$$

is easily computed using the methods of appendix B. To understand the order of the projection operators, note that

$$\begin{aligned}\mathrm{Tr}_{R_1''',R_2'''}\Big(\Gamma_R(\sigma)\Big) &= \sum_i \langle i, R_1'''|\Gamma_R(\sigma)|i, R_2'''\rangle \\ &= \mathrm{Tr}\left(\mathcal{N}^{-1} P_{R\to R_2'''}\Gamma_R(n,n-2) P_{R\to R_1'''}\Gamma_R(\sigma)\right),\end{aligned}$$

so that the row (column) index of the trace is column (row) index of the intertwiner respectively.

B. Restricted characters

Starting from S_n, define a chain of subgroups $\mathcal{G}_i$ $i = 1, \ldots, d$ as follows

$$\mathcal{G}_1 = \{\sigma \in S_n | \sigma(n) = n\} \tag{B.1}$$

$$\mathcal{G}_i = \{\sigma \in \mathcal{G}_{i-1} | \sigma(n-i+1) = n-i+1\}, \qquad i = 2, 3, \ldots, d. \tag{B.2}$$

In this appendix we will give a simple algorithm for the computation of

$$\chi_{R_1,R_2}\Big((p_1,p_2,\ldots,p_m)\Big) \equiv \mathrm{Tr}_{R_1,R_2}\Big(\Gamma_R\big((p_1,p_2,\ldots,p_m)\big)\Big)$$

with R_1 and R_2 irreps of $\mathcal{G}_d$ subduced from R, $(p_1, p_2, \ldots, p_m)$ is an element of S_n specified using the cycle notation and $n - d < p_i \leq n$ $\forall i$. We call χ_{R_1,R_2} a *restricted character*. If $R_1 = R_2$, we will simply write χ_{R_1}. We have already seen that restricted characters

determine the normalization of the intertwiners. Further, they are also needed in the derivation of the hopping identities that determine the interactions between strings and the branes to which they are attached.

The first subsection of this appendix reviews the subgroup swap rule in the setting of a specific example. In the next subsection we will derive the algorithm for the computation of the restricted character. The third subsection of this appendix describes a graphical notation which considerably simplifies the computation. The remainder of the appendix then develops this diagrammatic notation further.

B.1 Review of the subgroup swap rule

In this appendix, we review the subgroup swap rule. The reader requiring a more detailed explanation can consult appendix D of [30]. Consider the restricted Schur polynomial

$$\chi^{(2)}_{R,R''}\Big|_1\Big|_2 = \frac{1}{(n-2)!}\sum_{\sigma\in S_n} \mathrm{Tr}_{R''}\left(\Gamma_R(\sigma)\right) Z^{i_1}_{i_{\sigma(1)}}\cdots Z^{i_{n-2}}_{i_{\sigma(n-2)}} (W^{(2)})^{i_{n-1}}_{i_{\sigma(n-1)}} (W^{(1)})^{i_n}_{i_{\sigma(n)}}.$$

The labelling on the left hand side tells us to first restrict with respect to the subgroup that leaves the index of $W^{(1)}$ inert, and then with respect to the subgroup that leaves the index of $W^{(2)}$ inert. In general, we will get a different polynomial if we were to restrict first with respect to the subgroup that leaves the index of $W^{(2)}$ inert, and then with respect to the subgroup that leaves the index of $W^{(1)}$ inert. There is a relation between these two sets of polynomials, which is known as the "subgroup swap rule".

We use the weights of the boxes of the Young diagrams in the subgroup swap rule. All weights are defined by the Young diagram before the swap. The weight of the box labelled with upper index 1 is denoted by c^U_1 and the weight of the box labelled with lower index 1 is denoted by c^L_1. Similarly for index 2. The upper and lower no-swap factors are given by

$$N_U = \sqrt{1-\frac{1}{(c^U_1-c^U_2)^2}}, \qquad N_L = \sqrt{1-\frac{1}{(c^L_1-c^L_2)^2}}.$$

The upper and lower swap factors are given by

$$S_U = \frac{1}{c^U_1-c^U_2}, \qquad S_L = \frac{1}{c^L_1-c^L_2}.$$

Our example uses a restricted Schur with three strings attached. Swapping strings 2 and 3, the subgroup swap rule gives

$$\chi_{\begin{array}{|c|c|c|}\hline \ & \ & 1\\2\\ \hline 2\\3\\ \hline 3\\1\\ \hline\end{array}}(\sigma)\Big|_1\Big|_2\Big|_3 = \left[N_L N_U \chi_{\begin{array}{|c|c|c|}\hline \ & \ & 1\\2\\ \hline 2\\3\\ \hline 3\\1\\ \hline\end{array}}(\sigma) + S_U N_L \chi_{\begin{array}{|c|c|c|}\hline \ & \ & 1\\2\\ \hline 3\\ \hline 2\\1\\ \hline\end{array}}(\sigma)\right]\Big|_1\Big|_3\Big|_2$$
$$+\left[S_L N_U \chi_{\begin{array}{|c|c|c|}\hline \ & \ & 1\\2\\ \hline 2\\1\\ \hline 3\\ \hline\end{array}}(\sigma) + S_U S_L \chi_{\begin{array}{|c|c|c|}\hline \ & \ & 1\\2\\ \hline 3\\1\\ \hline 2\\3\\ \hline\end{array}}(\sigma)\right]\Big|_1\Big|_3\Big|_2$$

where

$$N_U = \sqrt{1 - \frac{1}{(c_2^U - c_3^U)^2}} = \frac{\sqrt{3}}{2}$$
$$N_L = \sqrt{1 - \frac{1}{(c_2^L - c_3^L)^2}} = \frac{\sqrt{3}}{2},$$
$$S_U = \frac{1}{c_2^U - c_3^U} = \frac{1}{2},$$
$$S_L = \frac{1}{c_2^L - c_3^L} = \frac{1}{2}.$$

Whenever an index is swapped, we include a swap factor, and whenever there is no swap, we include a no-swap factor.

B.2 Computing restricted characters

Consider an irrep R of S_n labeled by a Young diagram which has at least two boxes, either of which can be dropped to leave a valid Young diagram. Label these two boxes by 1 and 2. Denote the weights of these boxes by c_1 and c_2. Denote the irrep of S_{n-2} obtained by dropping box 1 and then box 2 by R_1''. Denote the irrep of S_{n-2} obtained by dropping box 2 and then box 1 by R_2''. Our first task is to compute

$$\mathrm{Tr}_{R_1'',R_2''}\left(\Gamma_R\left((n,n-1)\right)\right).$$

Using the subgroup swap rule obtained in [30], we can write

$$\chi_{R_1''}\left((n,n-1)\right) = \left[1 - \frac{1}{(c_1-c_2)^2}\right]\chi_{R_2''}\left((n,n-1)\right) + \frac{1}{(c_1-c_2)^2}\chi_{R_1''}\left((n,n-1)\right) \quad \text{(B.3)}$$
$$+\sqrt{1-\frac{1}{(c_1-c_2)^2}}\,\frac{1}{c_1-c_2}\left[\chi_{R_1'',R_2''}\left((n,n-1)\right)+\chi_{R_2'',R_1''}\left((n,n-1)\right)\right].$$

A second application of the subgroup swap rule gives

$$\chi_{R_2'',R_1''}\left((n,n-1)\right) = \left[1 - \frac{1}{(c_1-c_2)^2}\right]\chi_{R_1'',R_2''}\left((n,n-1)\right) + \frac{1}{(c_1-c_2)^2}\chi_{R_2'',R_1''}\left((n,n-1)\right)$$
$$+\sqrt{1-\frac{1}{(c_1-c_2)^2}}\,\frac{1}{c_1-c_2}\left[\chi_{R_2''}\left((n,n-1)\right)-\chi_{R_1''}\left((n,n-1)\right)\right]. \quad \text{(B.4)}$$

Now, substituting the results [30]

$$\chi_{R_1''}\left((n,n-1)\right) = \frac{1}{c_1-c_2}\mathrm{dim}_{R_1''}, \qquad \chi_{R_2''}\left((n,n-1)\right) = \frac{1}{c_2-c_1}\mathrm{dim}_{R_2''},$$

into (B.3) and (B.4) and solving, we obtain

$$\chi_{R_1'',R_2''}\left((n,n-1)\right) = \sqrt{1-\frac{1}{(c_1-c_2)^2}}\mathrm{dim}_{R_1''} = \chi_{R_2'',R_1''}\left((n,n-1)\right).$$

Next, consider an irrep of S_n labeled by Young diagram R . Choose three boxes in this Young diagram, and label them 1, 2 and 3 respectively. Choose the boxes so that dropping box 1 gives a legal Young diagram R' labeling an irrep of S_{n-1}, dropping box 1 and then box 2 gives a legal Young diagram R'' labeling an irrep of S_{n-2}, and dropping box 1, then box 2 and then box 3 again gives a legal Young diagram R''' labeling an irrep of S_{n-3}. We will compute

$$\chi_{R'''}\left((n,n-2)\right) = \mathrm{Tr}_{R'''}\left(\Gamma_R\left((n,n-2)\right)\right).$$

In what follows, we will frequently need to refer to vectors belonging to the carrier spaces of specific representations subduced by R when boxes are dropped from R. A convenient notation is to list the labels of the boxes that must be dropped from R in the order in which they must be dropped. Thus, the ket $|i,123\rangle$ is the i^{th} ket belonging to the carrier space of the S_{n-3} irrep obtained by dropping box 1, then box 2 and then box 3 from R; the ket $|j,231\rangle$ is the j^{th} ket belonging to the carrier space of the S_{n-3} irrep obtained by dropping box 2, then box 3 and then box 1 from R (assuming of course that the boxes can be dropped from R in this order, giving a legal Young diagram at each step). Start by writing

$$\begin{aligned}\chi_{R'''}((n,n-2)) &= \sum_{i=1}^{\dim_{R'''}} \langle i,123|\Gamma_R\left((n,n-2)\right)|i,123\rangle \\ &= \sum_{i=1}^{\dim_{R'''}} \langle i,123|\Gamma_{R'}\left((n-1,n-2)\right)\Gamma_R\left((n,n-1)\right)\Gamma_{R'}\left((n-1,n-2)\right)|i,123\rangle.\end{aligned}$$

Noting that $\Gamma_{R'}\left((n-1,n-2)\right)|i,123\rangle$ must belong to the carrier space of R', and using the completeness relation ($1_{R'}$ is the identity on the R' carrier space)

$$1_{R'} = \sum_{k=1}^{\dim_{R'}} |k,1\rangle\langle k,1|,$$

we have

$$\begin{aligned}\chi_{R'''}\left((n,n-2)\right) = \sum_{i=1}^{\dim_{R'''}}\sum_{j,k=1}^{\dim_{R'}} &\langle i,123|\Gamma_{R'}\left((n-1,n-2)\right)|k,1\rangle\langle k,1|\Gamma_R\left((n,n-1)\right)|j,1\rangle \\ &\times\langle j,1|\Gamma_{R'}\left((n-1,n-2)\right)|i,123\rangle.\end{aligned}$$

Now, decompose R' into a direct sum of S_{n-2} irreps $R' = \oplus R''_\beta$. Use the label β to denote the box that must be dropped from R' to obtain R''_β. Thus, we can write

$$1_{R'} = \sum_{k=1}^{\dim_{R'}} |k,1\rangle\langle k,1| = \sum_{\beta}\sum_{k=1}^{\dim_{R''_\beta}} |k,1\beta\rangle\langle k,1\beta|,$$

and hence

$$\begin{aligned}\chi_{R'''}\left((n,n-2)\right) = \sum_{i=1}^{\dim_{R'''}}\sum_{\beta_1,\beta_2}\sum_{k=1}^{\dim_{R''_{\beta_1}}}\sum_{j=1}^{\dim_{R''_{\beta_2}}} &\langle i,123|\Gamma_{R'}\left((n-1,n-2)\right)|k,1\beta_1\rangle \\ &\times\langle k,1\beta_1|\Gamma_R\left((n,n-1)\right)|j,1\beta_2\rangle\langle j,1\beta_2|\Gamma_{R'}\left((n-1,n-2)\right)|i,123\rangle.\end{aligned}$$

Now, introduce the operator $O(2)$ obtained by summing all two cycles of the S_{n-2} subgroup of which the R''_β are irreps. This operator is a Casimir of S_{n-2}. If the Young diagram R''_β has r_i boxes in the i^{th} row and c_i boxes in the i^{th} column, then when acting on the carrier space of R''_β we have [40]

$$O(2)|i,1\beta\rangle = \left[\sum_i \frac{r_i(r_i-1)}{2} - \sum_j \frac{c_j(c_j-1)}{2}\right]|i,1\beta\rangle \equiv \lambda_\beta|i,1\beta\rangle.$$

Clearly, for the problem we study here, $\lambda_{\beta_1} = \lambda_{\beta_2}$ if and only if R_{β_1} and R_{β_2} have the same shape as Young diagrams. From the definition of the $\mathcal{G}_2$ subgroup given above, it is clear that

$$\left[O(2), \Gamma_R\left((n,n-1)\right)\right] = 0.$$

It is now a simple matter to see that

$$\begin{aligned}\lambda_{\beta_1}\langle k,1\beta_1|\Gamma_R\left((n,n-1)\right)|j,1\beta_2\rangle &= \langle k,1\beta_1|O(2)\Gamma_R\left((n,n-1)\right)|j,1\beta_2\rangle \\ &= \langle k,1\beta_1|\Gamma_R\left((n,n-1)\right)O(2)|j,1\beta_2\rangle \\ &= \lambda_{\beta_2}\langle k,1\beta_1|\Gamma_R\left((n,n-1)\right)|j,1\beta_2\rangle\end{aligned}$$

so that $\langle k,1\beta_1|\Gamma_R\left((n,n-1)\right)|j,1\beta_2\rangle$ vanishes if R_{β_1} and R_{β_2} do not have the same shape. A completely parallel argument, using a Casimir of S_{n-3}, can be used to show that $\langle j,1\alpha_1\alpha_2|\Gamma_{R'}\left((n-1,n-2)\right)|i,123\rangle$ is only non-zero if $\alpha_1 = 2$, $\alpha_2 = 3$ or $\alpha_1 = 3$, $\alpha_2 = 2$. Thus,

$$\begin{aligned}\chi_{R'''}\left((n,n-2)\right) = \sum_{i=1,j,k}^{\dim_{R'''}} \Big[&\langle i,123|\Gamma_{R'}\left((n-1,n-2)\right)|k,123\rangle\langle k,123|\Gamma_R\left((n,n-1)\right)|j,123\rangle \\ &\times\langle j,123|\Gamma_{R'}\left((n-1,n-2)\right)|i,123\rangle + \langle i,123|\Gamma_{R'}\left((n-1,n-2)\right)|k,132\rangle \\ &\times\langle k,132|\Gamma_R\left((n,n-1)\right)|j,132\rangle\langle j,132|\Gamma_{R'}\left((n-1,n-2)\right)|i,123\rangle\Big] \\ = \left[\frac{1}{(c_2-c_3)^2}\frac{1}{c_1-c_2} + \left(1-\frac{1}{(c_2-c_3)^2}\right)\frac{1}{c_1-c_3}\right]&\dim_{R'''}.\end{aligned}$$

This example illustrates the general algorithm to be used to compute restricted characters:

- The group element whose trace is to be computed, can be decomposed into a product of two cycles of the form $\Gamma_R\left((i,i+1)\right)$. A complete set of states is inserted between each factor.
- Using appropriately chosen Casimirs, one can argue that the only non-zero matrix elements of each factor, are obtained when the order of boxes dropped to obtain the carrier space of the bra matches the order of boxes dropped to obtain the carrier space of the ket, except for the $(n-i+1)^{\text{th}}$ and $(n-i+2)^{\text{th}}$ boxes, whose order can be swapped.
- We can plug in the known value of the restricted character, which we have computed for precisely the two cases arising in the previous point.

B.3 Strand diagrams

Strand diagrams are a graphical notation designed to compute restricted characters. Strand diagrams keep track of two things:

- The order in which boxes are to be dropped and the identity (= position within the Young diagram) of the boxes.
- The group element whose trace we are computing.

If we are to drop n boxes, we draw a picture with n columns. The columns are populated by labeled strands - each strand represents one of the boxes that are to be dropped. We label the strands by the upper index in the box. The reader is strongly advised to read appendix E for a summary of our graphical notation. Whatever appears in the first column is to be dropped first; whatever appears in the second column is to be dropped second and so on. The strands are ordered at the top of the diagram, according to the order in which they must be dropped to get the row index. The strands are ordered at the bottom of the diagram according to the column index. The strands move from the top of the diagram to the bottom of the diagram, without breaking, so that strands ends at the top connect to the corresponding strand ends at the bottom. To connect the strands (which in general are in a different order at the top and bottom of the diagram) we need to weave the strands, thereby allowing them to swap columns. The allowed swaps depends on the specific group element whose trace we are computing. To determine the allowed swaps, write the group element as a product of cycles of the form $(i, i+1)$. For example, we would write

$$(n, n-2) = (n, n-1)(n-1, n-2)(n, n-1).$$

Each time we drop a box, we are considering a new subgroup. The action of the permutation group can be visualized as a permutation of n indices. The subgroups are obtained by considering elements that hold certain indices fixed (see (B.1) and (B.2)). Choose the subgroups involved so that when box i is dropped, $n-i+1$ is held fixed. Clearly then, each column j is associated with the index $n-j+1$. Each cycle $(i, i+1)$ is drawn as a box which straddles the columns associated with indices i and $i+1$. When the strands pass through a box, they may do so without swapping or by swapping columns. Each box is associated with a factor. Imagine that the strands passing through the box, reading from left to right, are labeled n and m. The weights associated with these boxes are c_n and c_m respectively. If the strands do not swap inside the box the factor for the box is

$$f_{\text{no swap}} = \frac{1}{c_n - c_m}.$$

If the strands do swap inside the box, the factor is

$$f_{\text{swap}} = \sqrt{1 - \frac{1}{(c_n - c_m)^2}}.$$

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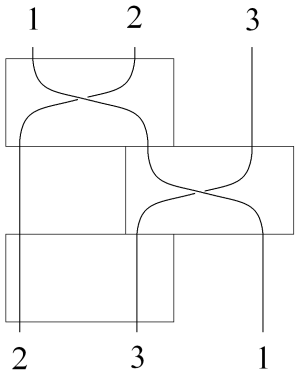

Figure 3: The strand diagram used in the computation of χ_1.

Denote the product of the factors, one from each box, by F. We have

$$\mathrm{Tr}_{R_1,R_2}\Big(\Gamma_R(\sigma)\Big) = \sum_i F_i \mathrm{dim}_{R_1},$$

where the index i runs over all possible paths consistent with the boundary conditions. With a little thought, the astute reader should be able to convince herself that this graphical rule is nothing but a convenient representation of the computation of the last subsection.

B.4 Strand diagram examples

In this section we will illustrate the use of strand diagrams in the computation of restricted characters. For our first example, we consider the computation of

$$\chi_1 = \mathrm{Tr}_{\begin{array}{|c|c|}\hline \ & \ \\ \hline 2 & 1 \\ \hline 3 & \\ \cline{1-1}\end{array}}\Big(\Gamma_{\ydiagram{3,2,1}}\big((6,4)\big)\Big).$$

Writing $(6,4) = (6,5)(4,5)(6,5)$ we obtain the strand diagram shown in figure 3. The factors for the upper most, middle and lower most boxes are

$$\sqrt{1-\frac{1}{(c_1-c_2)^2}}, \qquad \sqrt{1-\frac{1}{(c_1-c_3)^2}}, \qquad \frac{1}{c_2-c_3}$$

respectively. Thus,

$$\begin{aligned}\chi_1 &= \sqrt{1-\frac{1}{(c_1-c_2)^2}}\sqrt{1-\frac{1}{(c_1-c_3)^2}}\frac{1}{c_2-c_3}\mathrm{dim}\ydiagram{2,1} \\ &= 2\sqrt{1-\frac{1}{(c_1-c_2)^2}}\sqrt{1-\frac{1}{(c_1-c_3)^2}}\frac{1}{c_2-c_3}.\end{aligned}$$

The alert reader may worry that our recipe is not unique. Indeed we could also have written $(6,4) = (4,5)(6,5)(4,5)$. In this case, we obtain the strand diagram given in

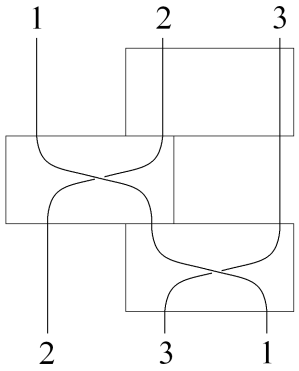

Figure 4: A second strand diagram that can be used in the computation of χ_1.

figure 4. In this case, the factors for the upper most, middle and lower most boxes are

$$\frac{1}{c_2 - c_3}, \qquad \sqrt{1 - \frac{1}{(c_1 - c_2)^2}}, \qquad \sqrt{1 - \frac{1}{(c_1 - c_3)^2}}$$

respectively. This gives exactly the same value for χ_1.

Next, we consider the computation of

$$\chi_2 = \mathrm{Tr}_{\begin{array}{|c|c|c|}\hline 1\\\hline 2\\\hline 3\\\hline\end{array}}\Big(\Gamma_{\begin{array}{|c|c|c|}\hline\\\hline\\\hline\\\hline\end{array}}((6,4))\Big).$$

This example is interesting as more than one path contributes. Writing $(6,4) = (4,5)(6,5)(4,5)$ we obtain the strand diagrams shown in figure 5. The product of factors for the diagram on the left is

$$\frac{1}{c_1 - c_3}\left[1 - \frac{1}{(c_2 - c_3)^2}\right].$$

The product of factors for the diagram on the right is

$$\frac{1}{c_1 - c_2}\frac{1}{(c_2 - c_3)^2}.$$

Thus,

$$\begin{aligned}\chi_2 &= \left(\frac{1}{c_1 - c_3}\left[1 - \frac{1}{(c_2 - c_3)^2}\right] + \frac{1}{c_1 - c_2}\frac{1}{(c_2 - c_3)^2}\right)\dim_{\begin{array}{|c|c|}\hline\\\hline\\\hline\end{array}} \\ &= 2\left(\frac{1}{c_1 - c_3}\left[1 - \frac{1}{(c_2 - c_3)^2}\right] + \frac{1}{c_1 - c_2}\frac{1}{(c_2 - c_3)^2}\right).\end{aligned}$$

The reader can check that the same value for χ_2 is obtained by decomposing $(6,4) = (6,5)(4,5)(6,5)$.

Finally, consider

$$\chi_3 = \mathrm{Tr}_{\begin{array}{|c|c|c|}\hline 1\\\hline 2\\\hline 3\\\hline\end{array}}\Big(\Gamma_{\begin{array}{|c|c|c|}\hline\\\hline\\\hline\\\hline\end{array}}((1))\Big).$$

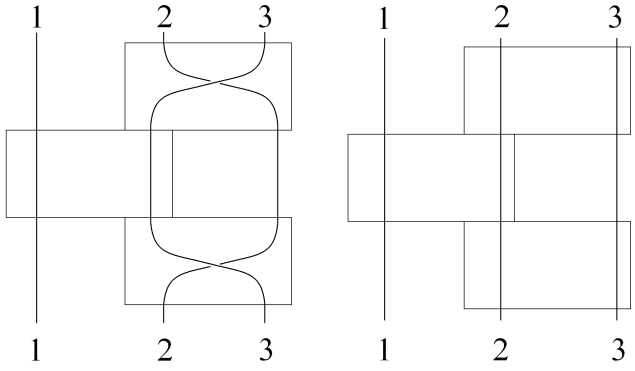

Figure 5: The strand diagrams used in the computation of χ_2.

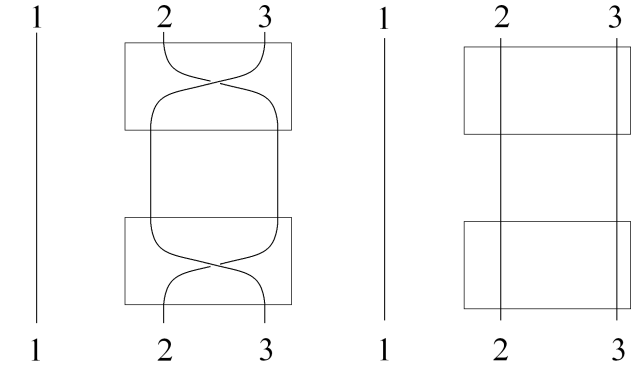

Figure 6: The strand diagrams used in the computation of χ_3.

Since we consider the identity element, the strand diagram has no boxes and hence $\chi_3 = \dim_{\square\!\square} = 2$. Since $(4,5)(4,5) = 1$ we could also have written

$$\chi_3 = \mathrm{Tr}_{\begin{array}{|c|c|c|}\hline 1\\\hline 2\\\hline 3\\\hline\end{array}}\Big(\Gamma_{\begin{array}{|c|c|c|}\hline\\\hline\\\hline\\\hline\end{array}}\big((4,5)(4,5)\big)\Big).$$

In this case there are two strand diagrams given in figure 6. Summing the contributions from these two strand diagrams we obtain

$$\chi_3 = \frac{1}{(c_2-c_3)^2}\dim_{\begin{array}{|c|c|}\hline\\\hline\\\hline\end{array}} + \left(1-\frac{1}{(c_2-c_3)^2}\right)\dim_{\begin{array}{|c|c|}\hline\\\hline\end{array}} = \dim_{\begin{array}{|c|c|}\hline\\\hline\end{array}} = 2.$$

Once again, the two ways of writing the restricted character give the same result. Note that the trace

$$\chi_3 = \mathrm{Tr}_{\begin{array}{|c|c|c|}\hline 1\\\hline 2\\\hline 3\\\hline\end{array}}\Big(\Gamma_{\begin{array}{|c|c|c|}\hline\\\hline\\\hline\\\hline\end{array}}\big((1)\big)\Big),$$

clearly vanishes because we are tracing the identity over an off the diagonal block. This is reflected graphically by the fact that there is no strand diagram that can be drawn -

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the order of strands at the top of the diagram does not match the order of strands at the bottom of the diagram and since we consider the identity element, the strand diagram has no boxes.

B.5 Tests of the restricted character results

By summing well chosen restricted characters, one can recover the characters of S_n which are known. This provides a number of tests that our restricted character formulas pass. As an example, consider the computation of $\chi_R\left((6,7)\right)$ for

$$R = \begin{smallmatrix}\square&\square&\square&\square&\square&\square\\ \square\end{smallmatrix}\,.$$

From the character tables for S_7 we find $\chi_R\left((6,7)\right) = 4$. In terms of restricted characters

$$\chi_R\left((6,7)\right) = \chi_{\begin{smallmatrix}\square&\square&\square&\square&2&1\\ \square\end{smallmatrix}}\left((6,7)\right) + \chi_{\begin{smallmatrix}\square&\square&\square&\square&\square&1\\ 2\end{smallmatrix}}\left((6,7)\right) + \chi_{\begin{smallmatrix}\square&\square&\square&\square&\square&2\\ 1\end{smallmatrix}}\left((6,7)\right).$$

Using the algorithm given above, it is straight forward to verify that

$$\chi_{\begin{smallmatrix}\square&\square&\square&\square&2&1\\ \square\end{smallmatrix}}\left((6,7)\right) = \dim_{\begin{smallmatrix}\square&\square&\square&\square\\ \square\end{smallmatrix}} = 4,$$

$$\chi_{\begin{smallmatrix}\square&\square&\square&\square&\square&1\\ 2\end{smallmatrix}}\left((6,7)\right) = \frac{1}{6}, \quad \chi_{\begin{smallmatrix}\square&\square&\square&\square&\square&2\\ 1\end{smallmatrix}}\left((6,7)\right) = -\frac{1}{6},$$

which do indeed sum to give 4. The reader is invited to check some more examples herself.

As a further check of our methods, we have computed the restricted characters $\mathrm{Tr}_{R_1,R_2}\left(\Gamma_R\left[\sigma\right]\right)$ numerically. This was done by explicitly constructing the matrices $\Gamma_R\left[\sigma\right]$. Each representation used was obtained by induction. One induces a reducible representation; the irreducible representation that participates was isolated using projection operators built from the Casimir obtained by summing over all two cycles. See appendix B.2 of [30] for more details. The resulting irreducible representations were tested by verifying the multiplication table of S_n. The intertwiners were computed using the projection operators of [30] and the results of appendix A; the normalization of the intertwiner was computed numerically.

B.6 Representations of S_n from strand diagrams

Using Strand diagrams, it is possible to write down the irreducible matrix representations of S_n. We will treat the simplest nontrivial example of S_3. First consider the $\begin{smallmatrix}\square&\square\\ \square\end{smallmatrix}$ irrep. Start by numbering the boxes in the Young diagram labeling the irrep, with an ordering in which the boxes are to be removed, so that one is left with a legal Young diagram after each box is removed. These labeled Young diagrams are in one-to-one correspondence with the matrix indices of the matrices in the irrep. For our example,

$$i = 1, \leftrightarrow \begin{smallmatrix}3&1\\ 2\end{smallmatrix} \qquad\qquad i = 2, \leftrightarrow \begin{smallmatrix}3&2\\ 1\end{smallmatrix}\,.$$

Each matrix element of $\Gamma_{\begin{smallmatrix}\square\square\\\square\end{smallmatrix}}((12))$ is given by a single strand diagram

$$\left[\Gamma_{\begin{smallmatrix}\square\square\\\square\end{smallmatrix}}((12))\right]_{11} = \mathrm{Tr}_{\begin{smallmatrix}3&1\\2\end{smallmatrix}}((12)) = \frac{1}{c_1-c_2} = \frac{1}{2},$$

$$\left[\Gamma_{\begin{smallmatrix}\square\square\\\square\end{smallmatrix}}((12))\right]_{12} = \mathrm{Tr}_{\begin{smallmatrix}3&1\\2\\1\end{smallmatrix}}((12)) = \sqrt{1-\frac{1}{(c_1-c_2)^2}} = \frac{\sqrt{3}}{2},$$

$$\left[\Gamma_{\begin{smallmatrix}\square\square\\\square\end{smallmatrix}}((12))\right]_{21} = \mathrm{Tr}_{\begin{smallmatrix}3&2\\1\\2\end{smallmatrix}}((12)) = \sqrt{1-\frac{1}{(c_1-c_2)^2}} = \frac{\sqrt{3}}{2},$$

and

$$\left[\Gamma_{\begin{smallmatrix}\square\square\\\square\end{smallmatrix}}((12))\right]_{22} = \mathrm{Tr}_{\begin{smallmatrix}3&2\\1\end{smallmatrix}}((12)) = \frac{1}{c_1-c_2} = -\frac{1}{2},$$

so that

$$\Gamma_{\begin{smallmatrix}\square\square\\\square\end{smallmatrix}}((12)) = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}.$$

In exactly the same way we obtain

$$\Gamma_{\begin{smallmatrix}\square\square\\\square\end{smallmatrix}}((23)) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}.$$

These two elements can now be used to generate the complete irrep.

Next consider $\square\square\square$. There is only one valid labeling $\boxed{3}\boxed{2}\boxed{1}$, so that the representation is one dimensional. It is straight forward to obtain

$$\mathrm{Tr}_{\boxed{3}\boxed{2}\boxed{1}}((12)) = \frac{1}{c_1-c_2} = 1, \qquad \mathrm{Tr}_{\boxed{3}\boxed{2}\boxed{1}}((23)) = \frac{1}{c_2-c_3} = 1,$$

which are the correct results. Finally, consider $\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$. Again, there is only one valid labeling so that the representation is again one dimensional. We find

$$\mathrm{Tr}_{\begin{smallmatrix}3\\2\\1\end{smallmatrix}}((12)) = \frac{1}{c_1-c_2} = -1, \qquad \mathrm{Tr}_{\begin{smallmatrix}3\\2\\1\end{smallmatrix}}((23)) = \frac{1}{c_2-c_3} = -1,$$

which are again the correct results.

C. Hopping identity

In this appendix, we derive identities that can be used to obtain the Cuntz chain Hamiltonian that accounts for the $O(g_{\mathrm{YM}}^2)$ correction to the anomalous dimension of our operators. To construct the "hop off" process, we use the fact that whenever a Z field hops past the borders of the open string word W, the resulting restricted Schur polynomial decomposes

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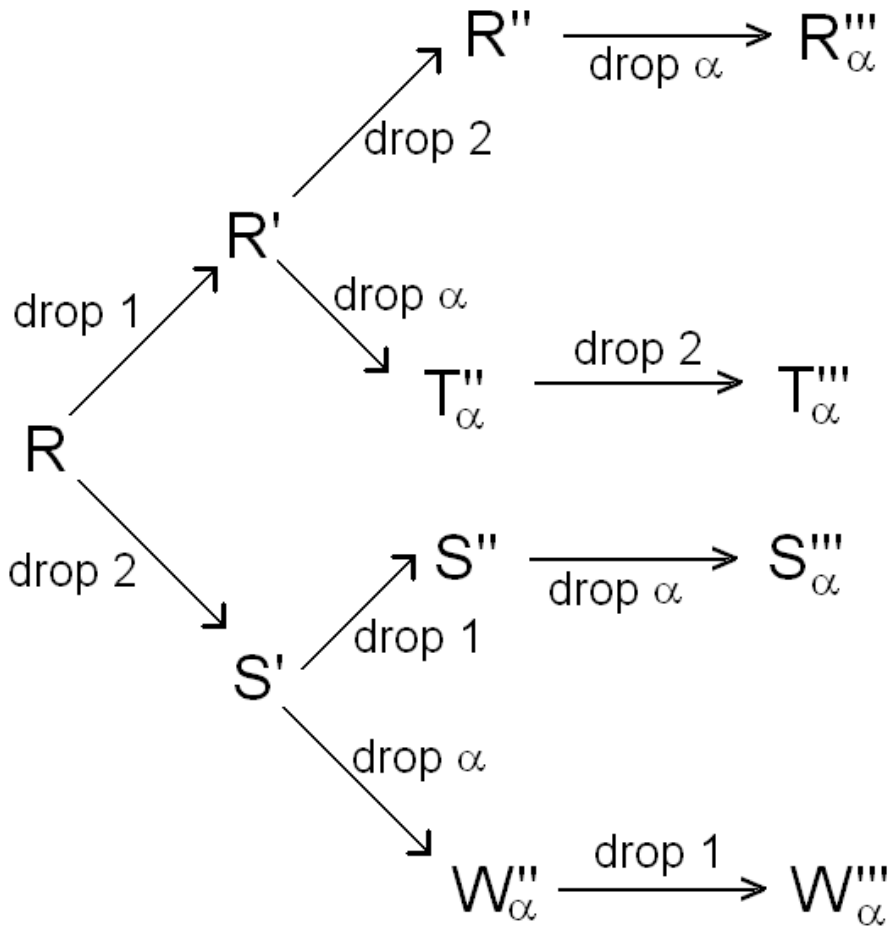

Figure 7: This figure shows the irreps that are used in the hopping identities. Starting from R, the figure shows which irrep is obtained when boxes in R are dropped.

into a sum of two types of systems, one is a giant with a closed string and another is a string-giant system where the giant is now bigger. In the large N limit only the second type needs to be considered. The identities we derive in this appendix express this decomposition. The irreps which play a role in the derivation of the identities are illustrated in figure 7. The basic structure of the derivation of these identities is very similar. For this reason, we explicitly derive an identity in the next subsection and simply state the remaining identities. In contrast to the case of a single string attached [32], here it does make a difference if the first or last sites of the string participate in the hopping. The identities needed in these two cases are listed separately. We have performed extensive numerical checks of the identities, which we describe next. Finally, we explain how to express the leading large N form of the identities, in terms of states of the Cuntz chain.

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C.1 Derivation of a Hopping identity

Our starting point is the restricted Schur polynomial

$$\chi^{(2)}_{R,R''}\Big|_1\Big|_2 = \frac{1}{(n-2)!}\sum_{\sigma\in S_n} \mathrm{Tr}_{R''}\left(\Gamma_R(\sigma)\right) Z^{i_1}_{i_{\sigma(1)}}\cdots Z^{i_{n-2}}_{i_{\sigma(n-2)}} (W^{(2)})^{i_{n-1}}_{i_{\sigma(n-1)}} (W^{(1)})^{i_n}_{i_{\sigma(n)}}.$$

There are two labeled boxes in R; dropping box 1 gives irrep R'; dropping box 2 gives irrep R''. Since R' is an irrep of the S_{n-1} subgroup $\mathcal{G}_1 = \{\sigma \in S_n | \sigma(n) = n\}$, we say that the open string described by the word $W^{(1)}$ is associated to box 1. Since R'' is an irrep of the S_{n-2} subgroup $\mathcal{G}_2 = \{\sigma \in \mathcal{G}_1 | \sigma(n-1) = n-1\}$, we say that the open string described by the word $W^{(2)}$ is associated with box 2. Notice that, in the chain of subductions used to define the restricted Schur polynomial, the box associated with $W^{(1)}$ is dropped before the box associated to $W^{(2)}$. We have indicated this with the notation $\Big|_1\Big|_2$. Rewrite the sum over S_n as a sum over $\mathcal{G}_1$ and its cosets

$$\begin{aligned}\chi^{(2)}_{R,R''}(Z,W^{(1)},W^{(2)})\Big|_1\Big|_2 = \frac{1}{(n-2)!}\sum_{\sigma\in\mathcal{G}_1}\Big[&\mathrm{Tr}_{R''}(\Gamma_{R'}(\sigma))\, Z^{i_1}_{i_{\sigma(1)}}\cdots Z^{i_{n-2}}_{i_{\sigma(n-2)}} (W^{(2)})^{i_{n-1}}_{i_{\sigma(n-1)}} \mathrm{Tr}(W^{(1)}) \\ &+\mathrm{Tr}_{R''}\left(\Gamma_R((1,n)\sigma)\right)(W^{(1)}Z)^{i_1}_{i_{\sigma(1)}}\cdots Z^{i_{n-2}}_{i_{\sigma(n-2)}}(W^{(2)})^{i_{n-1}}_{i_{\sigma(n-1)}} + \cdots + \\ &+\mathrm{Tr}_{R''}\left(\Gamma_R((n-2,n)\sigma)\right) Z^{i_1}_{i_{\sigma(1)}}\cdots (W^{(1)}Z)^{i_{n-2}}_{i_{\sigma(n-2)}}(W^{(2)})^{i_{n-1}}_{i_{\sigma(n-1)}} + \\ &+\mathrm{Tr}_{R''}\left(\Gamma_R((n-1,n)\sigma)\right) Z^{i_1}_{i_{\sigma(1)}}\cdots Z^{i_{n-2}}_{i_{\sigma(n-2)}}((W^{(1)}W^{(2)})^{i_{n-1}}_{i_{\sigma(n-1)}}\Big].\end{aligned}$$

The first term on the right hand side is

$$\frac{1}{(n-2)!}\sum_{\sigma\in\mathcal{G}_1}\mathrm{Tr}_{R''}(\Gamma_{R'}(\sigma))\, Z^{i_1}_{i_{\sigma(1)}}\cdots Z^{i_{n-2}}_{i_{\sigma(n-2)}}(W^{(2)})^{i_{n-1}}_{i_{\sigma(n-1)}}\mathrm{Tr}\,(W^{(1)}) = \chi^{(1)}_{R',R''}(Z,W^{(2)})\mathrm{Tr}\,(W^{(1)}).$$

Using the methods of appendix B, we know that

$$\mathrm{Tr}_{R''}\left(\Gamma_R((n-1,n)\sigma)\right) = \frac{1}{c_1-c_2}\mathrm{Tr}_{R''}\left(\Gamma_{R'}(\sigma)\right),$$

so that the last term on the right hand side is

$$\begin{aligned}\frac{1}{(n-2)!}\sum_{\sigma\in\mathcal{G}_1}\mathrm{Tr}_{R''}\left(\Gamma_R((n,n-1)\sigma)\right) Z^{i_1}_{i_{\sigma(1)}}\cdots Z^{i_{n-2}}_{i_{\sigma(n-2)}}&(W^{(1)}W^{(2)})^{i_{n-1}}_{i_{\sigma(n-1)}} \\ &= \frac{1}{c_1-c_2}\chi^{(1)}_{R',R''}(Z,W^{(1)}W^{(2)}).\end{aligned}$$

Focus on the remaining terms on the right hand side. Each of these terms makes the same contribution. We need to evaluate

$$\mathrm{Tr}_{R''}\left(\Gamma_R((j,n)\sigma) = \sum_{i=1}^{\dim_{R''}}\langle i,12|\Gamma_R((j,n))\Gamma_{R'}(\sigma)|i,12\rangle.\right.$$

Using the techniques of appendix B, it is straight forward to show that (the sum on α in the next equation is a sum over all boxes that can be removed from R'' to leave a valid

Young diagram; the relevant S_{n-3} subgroup is given by $\{\sigma \in \mathcal{G}_2 | \sigma(j) = j\}$)

$$\begin{aligned}
\mathrm{Tr}_{R''}\left(\Gamma_R((j,n)\sigma\right) &= \sum_{\alpha} \sum_{i,k=1}^{\dim_{R'''_\alpha}} \langle i, 12\alpha|\Gamma_R((j,n))|k, 12\alpha\rangle\langle k, 12\alpha|\Gamma_{R'}(\sigma)|i, 12\alpha\rangle \\
&\quad + \sum_{\alpha} \sum_{i,k=1}^{\dim_{R'''_\alpha}} \langle i, 12\alpha|\Gamma_R((j,n))|k, 1\alpha 2\rangle\langle k, 1\alpha 2|\Gamma_{R'}(\sigma)|i, 12\alpha\rangle \\
&= \sum_{\alpha} \frac{1}{c_1 - c_\alpha}\left[1 + \frac{1}{(c_1-c_2)(c_2-c_\alpha)}\right] \mathrm{Tr}_{R'''_\alpha}(\Gamma_{R'}(\sigma)) \\
&\quad + \sum_{\alpha} \frac{1}{c_1-c_2}\frac{1}{c_1-c_\alpha}\sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\mathrm{Tr}_{T'''_\alpha, R'''_\alpha}(\Gamma_{R'}(\sigma)).
\end{aligned}$$

Thus, summing the remaining $n-2$ terms we obtain

$$\begin{aligned}
&\sum_{\alpha} \frac{1}{c_1 - c_\alpha}\left[1 + \frac{1}{(c_1-c_2)(c_2-c_\alpha)}\right] \chi^{(2)}_{R', R'''_\alpha}(Z, W^{(1)}Z, W^{(2)})\Big|_2\Big|_1 \\
&\qquad + \sum_{\alpha} \frac{1}{c_1-c_2}\frac{1}{c_1-c_\alpha}\sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\chi^{(2)}_{R'\to T'''_\alpha, R'''_\alpha}(Z, W^{(1)}Z, W^{(2)})\Big|_2\Big|_1.
\end{aligned}$$

A straight forward application of the subgroup swap rule gives

$$\begin{aligned}
\chi^{(2)}_{R', R'''_\alpha}(Z, W^{(1)}Z, W^{(2)})\Big|_2\Big|_1 &= \Bigg[\left(1 - \frac{1}{(c_2-c_\alpha)^2}\right)\chi^{(2)}_{R', T'''_\alpha}(Z, W^{(1)}Z, W^{(2)}) \\
+\frac{1}{(c_2-c_\alpha)^2}\chi^{(2)}_{R', R'''_\alpha}(Z, W^{(1)}Z, W^{(2)}) &+ \sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\frac{1}{c_2-c_\alpha}\Big(\chi^{(2)}_{R'\to R'''_\alpha T'''_\alpha}(Z, W^{(1)}Z, W^{(2)}) \\
&\qquad + \chi^{(2)}_{R'\to T'''_\alpha R'''_\alpha}(Z, W^{(1)}Z, W^{(2)})\Big)\Bigg]\Big|_1\Big|_2, \\
\chi^{(2)}_{R'\to T'''_\alpha R'''_\alpha}(Z, W^{(1)}Z, W^{(2)})\Big|_2\Big|_1 &= \Bigg[\left(1 - \frac{1}{(c_2-c_\alpha)^2}\right)\chi^{(2)}_{R'\to R'''_\alpha T'''_\alpha}(Z, W^{(1)}Z, W^{(2)}) \\
-\frac{1}{(c_2-c_\alpha)^2}\chi^{(2)}_{R'\to T'''_\alpha R'''_\alpha}(Z, W^{(1)}Z, W^{(2)}) &+ \sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\frac{1}{c_2-c_\alpha}\Big(\chi^{(2)}_{R', R'''_\alpha}(Z, W^{(1)}Z, W^{(2)}) \\
&\qquad - \chi^{(2)}_{R', T'''_\alpha}(Z, W^{(1)}Z, W^{(2)})\Big)\Bigg]\Big|_1\Big|_2.
\end{aligned}$$

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Thus, we finally obtain

$$\begin{aligned}\chi^{(2)}_{R,R''}(Z,W^{(1)},W^{(2)})\Big|_1\Big|_2 &= \chi^{(1)}_{R',R''}(Z,W^{(2)})\mathrm{Tr}\,(W^{(1)}) + \frac{1}{c_1-c_2}\chi^{(1)}_{R',R''}(Z,W^{(1)}W^{(2)}) \\ &+\sum_\alpha\left[\frac{1}{c_1-c_\alpha}\left(1-\frac{1}{(c_2-c_\alpha)^2}\right)\chi^{(2)}_{R',T'''_\alpha}(Z,W^{(1)}Z,W^{(2)})\right. \\ &+\frac{1}{c_1-c_2}\frac{1}{(c_2-c_\alpha)^2}\chi^{(2)}_{R',R'''_\alpha}(Z,W^{(1)}Z,W^{(2)}) \\ &+\frac{1}{c_1-c_2}\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\chi^{(2)}_{R'\to R'''_\alpha T'''_\alpha}(Z,W^{(1)}Z,W^{(2)}) \\ &\left.+\frac{1}{c_1-c_\alpha}\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\chi^{(2)}_{R'\to T'''_\alpha R'''_\alpha}(Z,W^{(1)}Z,W^{(2)})\right]\Big|_1\Big|_2.\end{aligned} \tag{C.1}$$

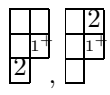
The above identity is relevant for interactions in which the impurity hops out of the last site of the string. For the hopping interaction in which the impurity hops out of the first site of the string, the right hand side of our identity should be written in terms of $ZW^{(1)}$. This identity is easily derived by rewriting the sum over S_n in terms of right cosets of $\mathcal{G}_1$ instead of left cosets as we have done above.

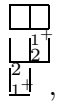
The identity derived above is relevant for the description of interactions in which string 1 exchanges momentum with the branes in the boundstate. To derive identities that allow string 2 to exchange momentum with the branes in the boundstate, we first use the subgroup swap rule to swap strings 1 and 2. We then rewrite the sum over S_n in terms of a sum over S_{n-1} and its cosets and then employ character identities as above. We give a complete set of identities in the next two subsections.

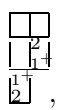
On first inspection, our identity (C.1) may appear intimidating. For this reason, we conclude this section with a concrete example of the use of our identity.

Consider for example $\chi^{(2)}_{R,R''} =$  . In C.1 the sum on α now yields one term for $\chi^{(2)}_{R',T'''_\alpha}(Z,W^{(1)}Z,W^{(2)})$, two terms for $\chi^{(2)}_{R',R'''_\alpha}(Z,W^{(1)}Z,W^{(2)})$ and one term for $\chi^{(2)}_{R'\to R'''_\alpha T'''_\alpha}(Z,W^{(1)}Z,W^{(2)})$ and $\chi^{(2)}_{R'\to T'''_\alpha R'''_\alpha}(Z,W^{(1)}Z,W^{(2)})$. Explicitly:

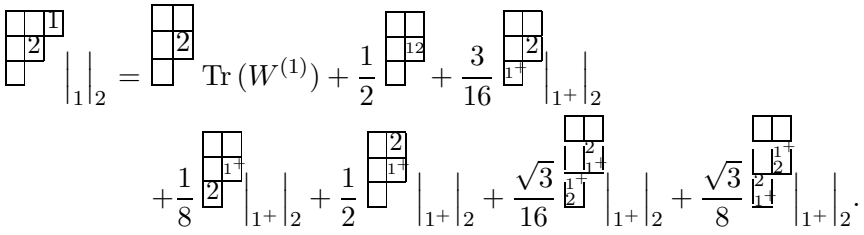
$$\chi^{(2)}_{R',T'''_\alpha}(Z,W^{(1)}Z,W^{(2)}) = \text{  },$$

$$\chi^{(2)}_{R',R'''_\alpha}(Z,W^{(1)}Z,W^{(2)}) = \text{  },$$

$$\chi^{(2)}_{R'\to R'''_\alpha T'''_\alpha}(Z,W^{(1)}Z,W^{(2)}) = \text{  },$$

$$\chi^{(2)}_{R'\to T'''_\alpha R'''_\alpha}(Z,W^{(1)}Z,W^{(2)}) = \text{  },$$

Now, $c_1 = \mathrm{N}+2$, $c_2 = \mathrm{N}$ and c_α (for a particular term in the identity) is equal to the weight of the labelled box in the restricted Schur polynomial of that term that does not correspond to either of the labelled boxes in the original Schur polynomial. The identity C.1 therefore becomes:



C.2 Identities relevant to Hopping off the first site of the string

$$\begin{aligned}\chi^{(2)}_{R,R''}(Z,W^{(1)},W^{(2)})\Big|_1\Big|_2 = {}& \chi^{(1)}_{R',R''}(Z,W^{(2)})\mathrm{Tr}\,(W^{(1)}) + \frac{1}{c_1-c_2}\chi^{(1)}_{R',R''}(Z,W^{(2)}W^{(1)}) \\ & + \sum_\alpha \left[\frac{1}{c_1-c_\alpha}\left(1-\frac{1}{(c_2-c_\alpha)^2}\right)\chi^{(2)}_{R',T''_\alpha}(Z,ZW^{(1)},W^{(2)})\right. \\ & + \frac{1}{c_1-c_2}\frac{1}{(c_2-c_\alpha)^2}\chi^{(2)}_{R',R''_\alpha}(Z,ZW^{(1)},W^{(2)}) \\ & + \frac{1}{c_1-c_2}\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\chi^{(2)}_{R'\to T''_\alpha R''_\alpha}(Z,ZW^{(1)},W^{(2)}) \\ & \left. + \frac{1}{c_1-c_\alpha}\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\chi^{(2)}_{R'\to R''_\alpha T''_\alpha}(Z,ZW^{(1)},W^{(2)})\right]\Big|_1\Big|_2\end{aligned} \tag{C.2}$$

The form of this identity is rather intuitive. The first term on the right hand side contributes to the process in which the bound state emits string 1; the second term describes the process in which the two open strings join to form one long open string. In both of these processes, the box which string 1 occupied on the left hand side does not appear on the right hand side. These two processes will not contribute to our Cuntz chain Hamiltonian; they are relevant for the description of interactions which change the number of open strings attached to the boundstate and do not contribute at the leading order of the large N expansion.

It is instructive to consider the form of this identity for well separated branes. For well separated branes, we have $|c_1 - c_2| \gg 1$. For $|c_1 - c_\alpha| \sim 1$, $|c_2 - c_\alpha| \gg 1$ so that of the last four terms only the first one contributes, giving $\approx \frac{1}{c_1-c_\alpha}\chi^{(2)}_{R',T''_\alpha}(Z,ZW^{(1)},W^{(2)})$. Thus, string 2 stays in box 2 and string 1 is close to where it started. Note that dropping terms of order $(c_1-c_2)^{-1}$ or $(c_\alpha-c_2)^{-1}$ we obtain

$$\chi^{(2)}_{R,R''}(Z,W^{(1)},W^{(2)})\Big|_1\Big|_2 \approx \chi^{(1)}_{R',R''}(Z,W^{(2)})\mathrm{Tr}\,(W^{(1)}) + \sum_\alpha \frac{1}{c_1-c_\alpha}\chi^{(2)}_{R',T''_\alpha}(Z,ZW^{(1)},W^{(2)}),$$

which is the identity of [32].

Next, consider the stretched string identities

$$\chi^{(2)}_{R\to R''S''}(\ Z\ ,W^{(1)},W^{(2)})\Big|_1\Big|_2 = \sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(1)}_{R',R''}(Z,W^{(2)}W^{(1)})$$
$$+\sum_\alpha\left[\frac{1}{c_1-c_\alpha}\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{R',R''_\alpha}(Z,ZW^{(1)},W^{(2)})\right. \tag{C.3}$$
$$\left.+\frac{1}{c_1-c_\alpha}\sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{R'\to T''_\alpha R''_\alpha}(Z,ZW^{(1)},W^{(2)})\right]\Big|_1\Big|_2$$

$$\chi^{(2)}_{R\to S''R''}(\ Z\ ,W^{(1)},W^{(2)})\Big|_1\Big|_2 = \sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(1)}_{S',S''}(Z,W^{(2)}W^{(1)})$$
$$+\sum_\alpha\left[\frac{1}{c_1-c_\alpha}\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{S',S''_\alpha}(Z,ZW^{(1)},W^{(2)})\right. \tag{C.4}$$
$$\left.+\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_1-c_\alpha)^2}}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{S'\to W''_\alpha S''_\alpha}(Z,ZW^{(1)},W^{(2)})\right]\Big|_1\Big|_2$$

Notice that in contrast to (C.2), (C.3) and (C.4) do not have a term on the right hand side corresponding to emission of string 1. This is what we would expect for an operator dual to a state with two strings stretching between branes, since if string 1 is emitted, it leaves a state with string 2 stretched between branes; this state is not allowed as it violates the Gauss Law. The process in which the two open strings join at their endpoints is allowed. In this process, it is the box with the upper 1 label that is removed. Thus, we can identify the Chan-Paton label for the side of the string defining the first lattice site of the Cuntz chain with the upper label for the string, in our diagrammatic notation. This corresponds to the first label of the restricted Schur polynomial. We will see further evidence for this interpretation when we interpret the final form of the Hamiltonian.

If we again consider the limit of two well separated branes, we find that (C.3) becomes

$$\chi^{(2)}_{R\to R''S''}(Z,W^{(1)},W^{(2)})\Big|_1\Big|_2 \approx \chi^{(1)}_{R',R''}(Z,W^{(2)}W^{(1)})$$
$$+\sum_\alpha\frac{1}{c_1-c_\alpha}\chi^{(2)}_{R'\to T''_\alpha R''_\alpha}(Z,ZW^{(1)},W^{(2)})\Big|_1\Big|_2.$$

In this case, the box with upper 1 label and lower 2 label moves from box 1 to box α (which are close to each other in the Young diagram) and box with upper 2 label and lower 1 label stays where it is.

The first three identities that we have discussed corresponded to an interaction in which an impurity from the first site of string 1 interacts with the brane. The next three identities that we discuss correspond to an interaction in which an impurity from the first

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site of string 2 interacts with the brane. The first three terms of the identity

$$\begin{aligned}\chi^{(2)}_{R,R''}(\ Z\ ,W^{(1)},W^{(2)})\Big|_1\Big|_2 &= \left(1-\frac{1}{(c_1-c_2)^2}\right)\chi^{(1)}_{S',S''}(Z,W^{(1)})\mathrm{Tr}\,(W^{(2)})\\ &+\frac{1}{(c_1-c_2)^2}\chi^{(1)}_{R',R''}(Z,W^{(1)})\mathrm{Tr}\,(W^{(2)})+\frac{1}{c_1-c_2}\chi^{(1)}_{R',R''}(Z,W^{(1)}W^{(2)})\\ &+\sum_\alpha\left[\frac{1}{c_2-c_\alpha}\left(1-\frac{1}{(c_1-c_2)^2}\right)\chi^{(2)}_{S',S'''_\alpha}(Z,W^{(1)},ZW^{(2)})\right.\\ &\quad+\frac{1}{c_2-c_\alpha}\frac{1}{(c_1-c_2)^2}\chi^{(2)}_{R',R'''_\alpha}(Z,W^{(1)},ZW^{(2)})\\ &\quad\left.+\frac{1}{c_1-c_2}\frac{1}{c_1-c_\alpha}\sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\chi^{(2)}_{R'\to R'''_\alpha T'''_\alpha}(Z,W^{(1)},ZW^{(2)})\right]\Big|_1\Big|_2\end{aligned}\tag{C.5}$$

change the number of open strings attached to the boundstate. The first two terms correspond to gravitational radiation; for both of these terms, string 2 is emitted as a closed string. The third term corresponds to a process in which the two open strings join to give a single open string. The order of the open string words in this term is not the same as the order in the corresponding term of (C.2). The term above is natural because it is the first site of string 2 that is interacting; the order in (C.2) also looks natural because in that case it is the first site of string 1 that is interacting. Notice that the above identity is rather different to (C.2). Physically this is surprising - since in both cases it is the first site of the string interacting, these identities should presumably look identical. This mismatch between the two identities is a consequence of the fact that we have treated string 1 and string 2 differently when constructing the operator. See section 3 for further discussion of this point.

If we again consider the limit of two well separated branes, we find that (C.5) becomes (take $|c_1-c_2|\gg 1$, $|c_1-c_\alpha|\gg 1$ and $|c_2-c_\alpha|\sim 1$)

$$\chi^{(2)}_{R,R''}(Z,W^{(1)},W^{(2)})\Big|_1\Big|_2 \approx \chi^{(1)}_{S',S''}(Z,W^{(1)})\mathrm{Tr}\,(W^{(2)})+\sum_\alpha\frac{1}{c_2-c_\alpha}\chi^{(2)}_{S',S'''_\alpha}(Z,W^{(1)},ZW^{(2)}).$$

This again reproduces the identity of [32]. Thus, the content of the formula for well separated branes matches the corresponding limit of (C.2). This is satisfying, because in this limit the order in which the strings are attached does not matter. This follows because the swap factor of [32] behaves as $|c_1-c_2|^{-1}$.

The remaining two identities are stretched string identities. In contrast to what we found above, there are terms corresponding to gravitational radiation in these identities. We interpret this as a signal that there is some mixing between the operators we have defined (which as explained above, made some arbitrary choices) to get to a "physical basis". See section 3 for more details. The first term in both identities

$$\begin{aligned}\chi^{(2)}_{R\to R''S''}(\ Z\ ,W^{(1)},W^{(2)})\Big|_1\Big|_2 &= \sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(1)}_{S',S''}(Z,W^{(1)}W^{(2)})\\ &+\frac{1}{c_1-c_2}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\left(\chi^{(1)}_{R',R''}(Z,W^{(1)})-\chi^{(1)}_{S',S''}(Z,W^{(1)})\right)\mathrm{Tr}\,(W^{(2)})\end{aligned}$$

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$$+\sum_\alpha \Bigg[\frac{1}{c_1-c_\alpha}\frac{1}{c_2-c_1}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{S',S''_\alpha}(Z,W^{(1)},ZW^{(2)}) \tag{C.6}$$
$$+\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\sqrt{1-\frac{1}{(c_\alpha-c_1)^2}}\chi^{(2)}_{S'\to S''_\alpha W''_\alpha}(Z,W^{(1)},ZW^{(2)})$$
$$+\frac{1}{c_1-c_2}\frac{1}{c_1-c_\alpha}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{R',R''_\alpha}(Z,W^{(1)},ZW^{(2)})\Bigg]\Big|_1\Big|_2,$$

$$\chi^{(2)}_{R\to S''R''}(\ Z\ ,W^{(1)},W^{(2)})\Big|_1\Big|_2=\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(1)}_{R',R''}(Z,W^{(1)}W^{(2)})$$
$$+\frac{1}{c_1-c_2}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\left(\chi^{(1)}_{R',R''}(Z,W^{(1)})-\chi^{(1)}_{S',S''}(Z,W^{(1)})\right)\mathrm{Tr}\,(W^{(2)})$$
$$+\sum_\alpha \Bigg[\frac{1}{c_2-c_\alpha}\frac{1}{c_1-c_2}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{R',R''_\alpha}(Z,W^{(1)},ZW^{(2)}) \tag{C.7}$$
$$+\frac{1}{c_1-c_\alpha}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\sqrt{1-\frac{1}{(c_\alpha-c_2)^2}}\chi^{(2)}_{R'\to R''_\alpha T''_\alpha}(Z,W^{(1)},ZW^{(2)})$$
$$+\frac{1}{c_2-c_1}\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{S',S''_\alpha}(Z,W^{(1)},ZW^{(2)})\Bigg]\Big|_1\Big|_2,$$

corresponds to two open strings joining to form one long open string. The order of the open string words in these terms again looks natural given that it is the first site of string 2 that is interacting. They will again not contribute in the leading order of the large N expansion. It is satisfying that the content of the large distance limit of (C.6)

$$\chi^{(2)}_{R\to R''S''}(Z,W^{(1)},W^{(2)})\Big|_1\Big|_2\approx\chi^{(1)}_{S',S''}(Z,W^{(1)}W^{(2)})+\sum_\alpha\frac{1}{c_2-c_\alpha}\chi^{(2)}_{S'\to S''_\alpha W''_\alpha}(Z,W^{(1)},ZW^{(2)}),$$

is in complete agreement with the large distance limit of (C.3).

C.3 Identities relevant to Hopping off the last site of the string

In this subsection, impurities hop between the last site of the strings and the threebrane. There are again six possible identities that we could consider. The first three identities de-

scribe an interaction between the last site of string 1 and the threebrane. The first identity

$$\begin{aligned}\chi^{(2)}_{R,R''}(\ Z\ ,W^{(1)},W^{(2)})\Big|_1\Big|_2 &= \chi^{(1)}_{R',R''}(Z,W^{(2)})\mathrm{Tr}\,(W^{(1)}) + \frac{1}{c_1-c_2}\chi^{(1)}_{R',R''}(Z,W^{(1)}W^{(2)}) \\ &+\sum_\alpha\left[\frac{1}{c_1-c_\alpha}\left(1-\frac{1}{(c_2-c_\alpha)^2}\right)\chi^{(2)}_{R',T'''_\alpha}(Z,W^{(1)}Z,W^{(2)})\right. \\ &\quad+\frac{1}{c_1-c_2}\frac{1}{(c_2-c_\alpha)^2}\chi^{(2)}_{R',R'''_\alpha}(Z,W^{(1)}Z,W^{(2)}) \\ &\quad+\frac{1}{c_1-c_2}\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\chi^{(2)}_{R'\to R'''_\alpha T'''_\alpha}(Z,W^{(1)}Z,W^{(2)}) \\ &\quad\left.+\frac{1}{c_1-c_\alpha}\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\chi^{(2)}_{R'\to T'''_\alpha R'''_\alpha}(Z,W^{(1)}Z,W^{(2)})\right]\Big|_1\Big|_2\end{aligned} \tag{C.8}$$

can be obtained from (C.2) by (i) swapping the labels on the twisted string states on the right hand side and (ii) swapping the order of the open string words in the second term on the right hand side. This is exactly what we would expect - it is now the last site of the string that is interacting; to swap the first and last sites, we must swap Chan-Paton indices i.e. we must swap the labels on the twisted string states. The discussion of this identity now parallels the discussion of (C.2) and is not repeated.

Consider next the stretched string identities

$$\begin{aligned}\chi^{(2)}_{R\to S''R''}(\ Z\ ,W^{(1)},W^{(2)})\Big|_1\Big|_2 &= \sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(1)}_{R',R''}(Z,W^{(1)}W^{(2)}) \\ &+\sum_\alpha\left[\frac{1}{c_1-c_\alpha}\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{R',R'''_\alpha}(Z,W^{(1)}Z,W^{(2)})\right. \\ &\quad\left.+\frac{1}{c_1-c_\alpha}\sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{R'\to R'''_\alpha T'''_\alpha}(Z,W^{(1)}Z,W^{(2)})\right]\Big|_1\Big|_2,\end{aligned} \tag{C.9}$$

$$\begin{aligned}\chi^{(2)}_{R\to R''S''}(\ Z\ ,W^{(1)},W^{(2)})\Big|_1\Big|_2 &= \sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(1)}_{S',S''}(Z,W^{(1)}W^{(2)}) \\ &+\sum_\alpha\left[\frac{1}{c_1-c_\alpha}\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{S',S'''_\alpha}(Z,W^{(1)}Z,W^{(2)})\right. \\ &\quad\left.+\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_1-c_\alpha)^2}}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{S'\to S'''_\alpha W'''_\alpha}(Z,W^{(1)}Z,W^{(2)})\right]\Big|_1\Big|_2.\end{aligned} \tag{C.10}$$

It is satisfying that identity (C.9) can be obtained from (C.3) and (C.10) from (C.4) by swapping the labels for stretched string states on both sides, and reversing the order of the open string words in the first term on the right hand side. The discussion of these identities now parallel the discussion of (C.3) and (C.4) and is not repeated.

The remaining three identities describe an interaction between the last site of string 2

and the threebrane. The identity

$$\begin{aligned}\chi^{(2)}_{R,R''}(\ Z\ ,W^{(1)},W^{(2)})\Big|_1\Big|_2 &= \left(1-\frac{1}{(c_1-c_2)^2}\right)\chi^{(1)}_{S',S''}(Z,W^{(1)})\mathrm{Tr}\,(W^{(2)})\\ &+\frac{1}{(c_1-c_2)^2}\chi^{(1)}_{R',R''}(Z,W^{(1)})\mathrm{Tr}\,(W^{(2)})+\frac{1}{c_1-c_2}\chi^{(1)}_{R',R''}(Z,W^{(2)}W^{(1)})\\ &+\sum_\alpha\left[\frac{1}{c_2-c_\alpha}\left(1-\frac{1}{(c_1-c_2)^2}\right)\chi^{(2)}_{S',S''_\alpha}(Z,W^{(1)},W^{(2)}Z)\right.\\ &\quad+\frac{1}{c_2-c_\alpha}\frac{1}{(c_1-c_2)^2}\chi^{(2)}_{R',R'''_\alpha}(Z,W^{(1)},W^{(2)}Z)\\ &\quad\left.+\frac{1}{c_1-c_2}\frac{1}{c_1-c_\alpha}\sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\chi^{(2)}_{R'\to T'''_\alpha R'''_\alpha}(Z,W^{(1)},W^{(2)}Z)\right]\Big|_1\Big|_2\end{aligned}\tag{C.11}$$

can be obtained from (C.5) by (i) swapping the labels on the twisted string states on the right hand side and (ii) swapping the order of the open string words in the second term on the right hand side. Finally, the stretched string identities

$$\begin{aligned}\chi^{(2)}_{R\to R''S''}(\ Z\ ,W^{(1)},W^{(2)})\Big|_1\Big|_2 &= \sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(1)}_{R',R''}(Z,W^{(2)}W^{(1)})\\ &+\frac{1}{c_1-c_2}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\left(\chi^{(1)}_{R',R''}(Z,W^{(1)})-\chi^{(1)}_{S',S''}(Z,W^{(1)})\right)\mathrm{Tr}\,(W^{(2)})\\ &+\sum_\alpha\left[\frac{1}{c_2-c_\alpha}\frac{1}{c_1-c_2}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{R',R'''_\alpha}(Z,W^{(1)},W^{(2)}Z)\right.\\ &\quad+\frac{1}{c_1-c_\alpha}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\sqrt{1-\frac{1}{(c_\alpha-c_2)^2}}\chi^{(2)}_{R'\to T'''_\alpha R'''_\alpha}(Z,W^{(1)},W^{(2)}Z)\\ &\quad\left.+\frac{1}{c_1-c_2}\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{S',S''_\alpha}(Z,W^{(1)},W^{(2)}Z)\right]\Big|_1\Big|_2\end{aligned}\tag{C.12}$$

$$\begin{aligned}\chi^{(2)}_{R\to S''R''}(\ Z\ ,W^{(1)},W^{(2)})\Big|_1\Big|_2 &= \sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(1)}_{S',S''}(Z,W^{(2)}W^{(1)})\\ &-\frac{1}{c_1-c_2}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\left(\chi^{(1)}_{S',S''}(Z,W^{(1)})-\chi^{(1)}_{R',R''}(Z,W^{(1)})\right)\mathrm{Tr}\,(W^{(2)})\\ &+\sum_\alpha\left[\frac{1}{c_1-c_\alpha}\frac{1}{c_2-c_1}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{S',S''_\alpha}(Z,W^{(1)},W^{(2)}Z)\right.\\ &\quad+\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\sqrt{1-\frac{1}{(c_\alpha-c_1)^2}}\chi^{(2)}_{S'\to W'''_\alpha S'''_\alpha}(Z,W^{(1)},W^{(2)}Z)\\ &\quad\left.+\frac{1}{c_1-c_2}\frac{1}{c_1-c_\alpha}\sqrt{1-\frac{1}{(c_1-c_2)^2}}\chi^{(2)}_{R',R'''_\alpha}(Z,W^{(1)},W^{(2)}Z)\right]\Big|_1\Big|_2\end{aligned}\tag{C.13}$$

can be obtained from (C.5) and (C.6) by swapping the labels for stretched string states

on both sides, and reversing the order of the open string words in the first term on the right hand side.

C.4 Numerical test

An important result of this article are the identities presented in the previous two subsections, since they determine the hop off interaction. The hop on interaction follows from the hop off interaction by Hermitian conjugation and the kissing interaction by composing the hop on and the hop off interactions. Thus, the complete boundary interaction and the corresponding back reaction on the brane are determined by these identities. For this reason, we have tested the identities numerically. In this subsection we will explain the check we have performed.

Our formulas are identities between restricted Schur polynomials. They must hold if we evaluate them for *any*[12] numerical value of the matrices Z and W. Our check entails evaluating our identities for randomly generated matrices $W^{(1)}$, $W^{(2)}$ and Z, to check their validity. Evaluating a restricted Schur polynomial entails evaluating a restricted character as well as a product of traces of a product of the matrices $W^{(1)}$, $W^{(2)}$ and Z.

The restricted character $\mathrm{Tr}_{R'',S''}\left(\Gamma_R[\sigma]\right)$ or $\mathrm{Tr}_{R''}\left(\Gamma_R[\sigma]\right)$ was computed by explicitly constructing the matrices $\Gamma_R[\sigma]$. Each representation used was obtained by induction. One induces a reducible representation; the irreducible representation that participates was isolated using projection operators built from the Casimir obtained by summing over all two cycles. See appendix B.2 of [30] for more details. The resulting irreducible representations were tested by verifying the multiplication table of S_n. The restricted trace is then evaluated with the help of a projection operator or an intertwiner. The intertwiner was computed using the results of appendix A.

The trace $\mathrm{Tr}\,(\sigma Z^{\otimes n-1}W^{(1)}W^{(2)}) = Z^{i_1}_{i_{\sigma(1)}}Z^{i_2}_{i_{\sigma(2)}}\cdots Z^{i_{n-2}}_{i_{\sigma(n-2)}}(W^{(2)})^{i_{n-1}}_{i_{\sigma(n-1)}}(W^{(1)})^{i_n}_{i_{\sigma(n)}}$ for any given $\sigma \in S_n$ is easily expressed as a product of traces of powers of Z, $W^{(1)}$ and $W^{(2)}$.

In total we verified over 50 specific instances of our identities, which provides a significant check of each identity.

C.5 Identities in terms of Cuntz chain states

The state-operator correspondence is available for any conformal field theory. Using this correspondence, we can trade our (local) operators for a set of states. Concretely, this involves quantizing with respect to radial time. Considering a fixed "radial time" slice we obtain a round sphere. The states dual to the restricted Schur polynomial operators are the states of our Cuntz chain. Thus, we need to rewrite the identities obtained in this appendix as statements in terms of the states of the Cuntz oscillator chain. The states of the Cuntz oscillator chain are normalized. Normalized states correspond to operators whose two point function is normalized. Using the technology of [30] it is a simple task to compute the free equal time correlators of the restricted Schur polynomials. After making use of the free field correlators to write our identities in terms operators with unit two point functions, we find that not all terms are of the same order in N. We drop all terms which

[12] In particular, not necessarily Hermitian.

are subleading in N. These terms are naturally interpreted in terms of string splitting and joining processes, so that they will be important when interactions that change the number of open strings are considered.

The discussion for all of the identities above is rather similar, so we will be content to discuss a specific example which illustrates the general features. Consider the right hand side of (C.2). From the equal time correlator (there are a total of h_i fields in open string word $W^{(i)}$; f_R is the product of the weights of the Young diagram R; d_R is the dimension of R as an irrep of the symmetric group; n_R is the number of boxes in Young diagram R)

$$\begin{aligned}&\langle\chi^{(1)}_{R',R''}(Z,W^{(2)})\mathrm{Tr}\,(W^{(1)})\chi^{(1)}_{R',R''}(Z,W^{(2)})^\dagger\mathrm{Tr}\,(W^{(1)})^\dagger\rangle\\&\qquad=\left(\frac{4\pi\lambda}{N}\right)^{h_1+h_2+n_{R''}}h_1N^{h_1+h_2-1}n_{R''}f_{R'}\frac{d_{R''}}{d_{R'}}\end{aligned}\tag{C.14}$$

we know that the operator $\chi^{(1)}_{R',R''}(Z,W^{(2)})\mathrm{Tr}\,(W^{(1)})$ corresponds to the state (all Cuntz chain states are normalized to 1)

$$\sqrt{\left(\frac{4\pi\lambda}{N}\right)^{h_1+h_2+n_{R''}}h_1N^{h_1+h_2-1}n_{R''}f_{R'}\frac{d_{R''}}{d_{R'}}}|R',R'',W^{(2)};W^{(1)}\rangle.$$

The result (C.14) is not exact. When computing $\langle\mathrm{Tr}\,(W^{(1)})\mathrm{Tr}\,(W^{(1)})^\dagger\rangle$ we have only summed the leading planar contribution. When computing $\langle\chi^{(1)}_{R',R''}(Z,W^{(2)})\chi^{(1)}_{R',R''}(Z,W^{(2)})^\dagger\rangle$ we have only kept the F_0 contribution in the language of [30]. We have also factorized $\langle\chi^{(1)}_{R',R''}(Z,W^{(2)})\mathrm{Tr}\,(W^{(1)})\chi^{(1)}_{R',R''}(Z,W^{(2)})^\dagger\mathrm{Tr}\,(W^{(1)})^\dagger\rangle$ as $\langle\chi^{(1)}_{R',R''}(Z,W^{(2)})\chi^{(1)}_{R',R''}(Z,W^{(2)})^\dagger\rangle\times\langle\mathrm{Tr}\,(W^{(1)})\mathrm{Tr}\,(W^{(1)})^\dagger\rangle$ which is valid at large N. Similarly, (again we sum only the leading order at large N)

$$\langle\chi^{(1)}_{R',R''}(Z,W^{(2)}W^{(1)})\chi^{(1)}_{R',R''}(Z,W^{(2)}W^{(1)})^\dagger\rangle=\left(\frac{4\pi\lambda}{N}\right)^{h_1+h_2+n_{R''}}N^{h_1+h_2-1}n_{R''}f_{R'}\frac{d_{R''}}{d_{R'}}$$

implies that $\chi^{(1)}_{R',R''}(Z,W^{(2)}W^{(1)})$ corresponds to the state

$$\sqrt{\left(\frac{4\pi\lambda}{N}\right)^{h_1+h_2+n_{R''}}N^{h_1+h_2-1}n_{R''}f_{R'}\frac{d_{R''}}{d_{R'}}}|R',R'',W^{(2)}W^{(1)})\rangle.$$

Finally, the correlators (again we sum only the leading order at large N)

$$\begin{aligned}&\langle\chi^{(2)}_{R',T''''_\alpha}(Z,ZW^{(1)},W^{(2)})\chi^{(2)}_{R',T''''_\alpha}(Z,ZW^{(1)},W^{(2)})^\dagger\rangle\\&\qquad=\left(\frac{4\pi\lambda}{N}\right)^{h_1+h_2+1+n_{T''''_\alpha}}N^{h_1+h_2-1}n^2_{R'}\frac{d_{T''''_\alpha}}{d_{R'}}f_{R'},\\&\langle\chi^{(2)}_{R',R''''_\alpha}(Z,ZW^{(1)},W^{(2)})\chi^{(2)}_{R',R''''_\alpha}(Z,ZW^{(1)},W^{(2)})^\dagger\rangle\\&\qquad=\left(\frac{4\pi\lambda}{N}\right)^{h_1+h_2+1+n_{T''''_\alpha}}N^{h_1+h_2-1}n^2_{R'}\frac{d_{R''''_\alpha}}{d_{R'}}f_{R'},\end{aligned}$$

$$\langle \chi^{(2)}_{R'\to T''''_\alpha R''''_\alpha}(Z,ZW^{(1)},W^{(2)})\chi^{(2)}_{R'\to T''''_\alpha R''''_\alpha}(Z,ZW^{(1)},W^{(2)})^\dagger\rangle$$
$$= \left(\frac{4\pi\lambda}{N}\right)^{h_1+h_2+1+n_{T''''_\alpha}} N^{h_1+h_2-1} n^2_{R'} \frac{d_{T''''_\alpha}}{d_{R'}} f_{R'},$$
$$\langle \chi^{(2)}_{R'\to R''''_\alpha T''''_\alpha}(Z,ZW^{(1)},W^{(2)})\chi^{(2)}_{R'\to R''''_\alpha T''''_\alpha}(Z,ZW^{(1)},W^{(2)})^\dagger\rangle$$
$$= \left(\frac{4\pi\lambda}{N}\right)^{h_1+h_2+1+n_{T''''_\alpha}} N^{h_1+h_2-1} n^2_{R'} \frac{d_{T''''_\alpha}}{d_{R'}} f_{R'}$$

imply the correspondences

$$\chi^{(2)}_{R',T''''_\alpha}(Z,ZW^{(1)},W^{(2)}) \longleftrightarrow \tag{C.15}$$
$$\sqrt{\left(\frac{4\pi\lambda}{N}\right)^{h_1+h_2+1+n_{T''''_\alpha}} N^{h_1+h_2-1} n^2_{R'} \frac{d_{T''''_\alpha}}{d_{R'}} f_{R'}}\,|R',T_{\alpha'''},ZW^{(1)},W^{(2)}\rangle,$$
$$\chi^{(2)}_{R',R''''_\alpha}(Z,ZW^{(1)},W^{(2)}) \longleftrightarrow \tag{C.16}$$
$$\sqrt{\left(\frac{4\pi\lambda}{N}\right)^{h_1+h_2+1+n_{T''''_\alpha}} N^{h_1+h_2-1} n^2_{R'} \frac{d_{R''''_\alpha}}{d_{R'}} f_{R'}}\,|R',R''''_\alpha,ZW^{(1)},W^{(2)}\rangle,$$
$$\chi^{(2)}_{R'\to T''''_\alpha R''''_\alpha}(Z,ZW^{(1)},W^{(2)}) \longleftrightarrow \tag{C.17}$$
$$\sqrt{\left(\frac{4\pi\lambda}{N}\right)^{h_1+h_2+1+n_{T''''_\alpha}} N^{h_1+h_2-1} n^2_{R'} \frac{d_{T''''_\alpha}}{d_{R'}} f_{R'}}\,|R',T''''_\alpha R''''_\alpha,ZW^{(1)},W^{(2)}\rangle$$
$$\chi^{(2)}_{R'\to R''''_\alpha T''''_\alpha}(Z,ZW^{(1)},W^{(2)}) \longleftrightarrow \tag{C.18}$$
$$\sqrt{\left(\frac{4\pi\lambda}{N}\right)^{h_1+h_2+1+n_{T''''_\alpha}} N^{h_1+h_2-1} n^2_{R'} \frac{d_{T''''_\alpha}}{d_{R'}} f_{R'}}\,|R',R''''_\alpha T''''_\alpha,ZW^{(1)},W^{(2)}\rangle$$

Consider the factor

$$n^2_{R'}\frac{d_{R''''_\alpha}}{d_{R'}} = \frac{(\text{hooks})_{R'}}{(\text{hooks})_{R''''_\alpha}},$$

where $(\text{hooks})_R$ is the product of the hook lengths of Young diagram R. It is straight forward to compute this ratio of hook lengths, which is generically of order N^2 implying that $\frac{d_{R''''_\alpha}}{d_{R'}}$ is of order 1. Using this observation, it is equally easy to verify that $\frac{d_{T''''_\alpha}}{d_{R'}}$ and $\frac{d_{R''}}{d_{R'}}$ are also both $O(1)$. Given these results, it is simple to see that the sum of operators

$$\chi^{(1)}_{R',R''}(Z,W^{(2)})\mathrm{Tr}\,(W^{(1)}) + \frac{1}{c_1-c_2}\chi^{(1)}_{R',R''}(Z,W^{(2)}W^{(1)})$$
$$+\sum_\alpha\Bigg[\frac{1}{c_1-c_\alpha}\left(1-\frac{1}{(c_2-c_\alpha)^2}\right)\chi^{(2)}_{R',T''''_\alpha}(Z,ZW^{(1)},W^{(2)})$$
$$+\frac{1}{c_1-c_2}\frac{1}{(c_2-c_\alpha)^2}\chi^{(2)}_{R',R''''_\alpha}(Z,ZW^{(1)},W^{(2)})$$
$$+\frac{1}{c_1-c_2}\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\chi^{(2)}_{R'\to T''''_\alpha R''''_\alpha}(Z,ZW^{(1)},W^{(2)})$$
$$+\frac{1}{c_1-c_\alpha}\frac{1}{c_2-c_\alpha}\sqrt{1-\frac{1}{(c_2-c_\alpha)^2}}\chi^{(2)}_{R'\to R''''_\alpha T''''_\alpha}(Z,ZW^{(1)},W^{(2)})\Bigg]\Big|_1\Big|_2$$

corresponds to the following sum of normalized states

$$\sqrt{\left(\frac{4\pi\lambda}{N}\right)^{h_1+h_2+n_{R''}} N^{h_1+h_2-1} n_{R'}^2 f_{R'}} \left[\sqrt{\frac{h_1 d_{R''}}{n_{R'} d_{R'}}} |R', R'', W^{(2)}; W^{(1)}\rangle \right.$$
$$\left. + \frac{1}{c_1 - c_2} \sqrt{\frac{d_{R''}}{n_{R'} d_{R'}}} |R', R'', W^{(2)} W^{(1)}\rangle \right.$$
$$+ \sum_\alpha \left[\frac{1}{c_1 - c_\alpha} \left(1 - \frac{1}{(c_2 - c_\alpha)^2}\right) \sqrt{\frac{d_{T'''_\alpha}}{d_{R'}}} |R', T'''_\alpha, Z W^{(1)}, W^{(2)}\rangle \right.$$
$$+ \frac{1}{c_1 - c_2} \frac{1}{(c_2 - c_\alpha)^2} \sqrt{\frac{d_{R'''_\alpha}}{d_{R'}}} |R', R'''_\alpha, Z W^{(1)}, W^{(2)}\rangle$$
$$+ \frac{1}{c_1 - c_2} \frac{1}{c_2 - c_\alpha} \sqrt{1 - \frac{1}{(c_2 - c_\alpha)^2}} \sqrt{\frac{d_{T'''_\alpha}}{d_{R'}}} |R', T'''_\alpha R'''_\alpha, Z W^{(1)}, W^{(2)}\rangle$$
$$\left. \left. + \frac{1}{c_1 - c_\alpha} \frac{1}{c_2 - c_\alpha} \sqrt{1 - \frac{1}{(c_2 - c_\alpha)^2}} \sqrt{\frac{d_{T'''_\alpha}}{d_{R'}}} |R', R'''_\alpha T'''_\alpha, Z W^{(1)}, W^{(2)}\rangle \right] \right] .$$

Recalling that $h_1 = O(\sqrt{N})$ and $n_{R'} = O(N)$, it is clear that the first two terms are subleading. These two terms correspond to gravitational radiation (first term) and string joining (second term); they are the only terms that correspond to an interaction that changes the number of open strings attached to the excited giant system. Although we have illustrated things with an example, this conclusion is general - for all of the identities obtained in this appendix, terms that do not correspond to two strings attached to the giant system can be dropped in the leading large N limit.

D. State/operator map

In this section we will simply quote the six normalization factors, in table 1, that enter the relation between the restricted Schur polynomials and the normalized Cuntz chain states relevant for the excited two giant graviton bound state.[13] The normalization factors are not exact - we simply quote the leading large N value of these normalizations. These factors are determined completely by the $F_0^{(1)} F_0^{(2)}$ contribution in the language of [30]. The factor f_R is the product of weights of the Young diagram R. The open string word $W^{(1)}$ contains a total number of h_1 Higgs fields; the open string word $W^{(2)}$ contains a total number of h_2 Higgs fields.

E. Review of the restricted Schur polynomial notation

In this appendix, we review the definition of the restricted Schur polynomial. The reader requiring more details can consult [20, 30, 32].

[13] See the introduction for the restricted Schur polynomials corresponding to these states.

State	Normalization
$\|b_0-1,b_1,11,22\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{2b_0+b_1+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}$
$\|b_0-1,b_1,22,11\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{2b_0+b_1+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}$
$\|b_0-1,b_1,12,21\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{2b_0+b_1+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}$
$\|b_0-1,b_1,21,12\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{2b_0+b_1+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}$
$\|b_0-2,b_1+2,22,22\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{2b_0+b_1+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{b_1+3}{b_1+1}}$
$\|b_0,b_1-2,11,11\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{2b_0+b_1+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{b_1-1}{b_1+1}}$

Table 1: Normalization of the Cuntz states dual to a two giant system.

There is by now convincing evidence that the dual of a giant graviton is a Schur polynomial. Schur polynomials are labeled by Young diagrams. Excitations of giant gravitons can be described by attaching open strings to the giant graviton. Operators dual to excitations of giant gravitons are obtained by inserting words $(W^{(a)})_i^j$ describing the open strings (one word for each open string) into the operator describing the system of giant gravitons

$$\chi^{(k)}_{R,R_1}(Z,W^{(1)},\ldots,W^{(k)}) = \frac{1}{(n-k)!}\sum_{\sigma\in S_n} \mathrm{Tr}_{R_1}(\Gamma_R(\sigma))\mathrm{Tr}\,(\sigma Z^{\otimes n-k}W^{(1)}\cdots W^{(k)}), \quad \text{(E.1)}$$

$$\mathrm{Tr}\,(\sigma Z^{\otimes n-k}W^{(1)}\cdots W^{(k)}) = Z^{i_1}_{i_{\sigma(1)}} Z^{i_2}_{i_{\sigma(2)}}\cdots Z^{i_{n-k}}_{i_{\sigma(n-k)}} (W^{(1)})^{i_{n-k+1}}_{i_{\sigma(n-k+1)}}\cdots (W^{(k)})^{i_n}_{i_{\sigma(n)}}. \quad \text{(E.2)}$$

The representation R of the giant graviton system is a Young diagram with n boxes, i.e. it is a representation of S_n. $\Gamma_R(\sigma)$ is the matrix representing σ in irreducible representation R of the symmetric group S_n. The representation R_1 is a Young diagram with $n-k$ boxes, i.e. it is a representation of S_{n-k}. Imagine that the k words above are all distinct, corresponding to the case that the open strings are distinguishable. Consider an $S_{n-k}\otimes (S_1)^k$ subgroup of S_n. The representation R of S_n will subduce a (generically) reducible representation of the $S_{n-k}\otimes(S_1)^k$ subgroup. One of the irreducible representations appearing in this subduced representation is R_1. Tr_{R_1} is an instruction to trace only over the indices belonging to this irreducible component. If the representation R_1 appears more than once, things are more interesting. The example discussed in [20] illustrates this point nicely. Suppose $R\to R_1\oplus R_2\oplus R_2$ under restricting S_n to $S_{n-2}\times S_1\times S_1$. Choose a basis so that

$$\Gamma_R(\sigma) = \begin{bmatrix} \Gamma_{R_1}(\sigma)_{i_1j_1} & 0 & 0 \\ 0 & \Gamma_{R_2}(\sigma)_{i_2j_2} & 0 \\ 0 & 0 & \Gamma_{R_2}(\sigma)_{i_3j_3} \end{bmatrix}, \qquad \forall\sigma\in S_{n-2}\times S_1\times S_1,$$

$$\Gamma_R(\sigma) = \begin{bmatrix} A^{(1,1)}_{i_1j_1} & A^{(1,2)}_{i_1j_2} & A^{(1,3)}_{i_1j_3} \\ A^{(2,1)}_{i_2j_1} & A^{(2,2)}_{i_2j_2} & A^{(2,3)}_{i_2j_3} \\ A^{(3,1)}_{i_3j_1} & A^{(3,2)}_{i_3j_2} & A^{(3,3)}_{i_3j_3} \end{bmatrix}, \qquad \sigma\notin S_{n-2}\times S_1\times S_1.$$

There are four suitable definitions for $\mathrm{Tr}_{R_2}(\Gamma_R(\sigma))$: $\mathrm{Tr}\,(A^{(2,2)})$, $\mathrm{Tr}\,(A^{(2,3)})$, $\mathrm{Tr}\,(A^{(3,2)})$ or $\mathrm{Tr}\,(A^{(3,3)})$. Interpret the operator obtained using $\mathrm{Tr}\,(A^{(2,3)})$ or $\mathrm{Tr}\,(A^{(3,2)})$ as dual to the

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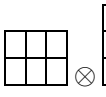
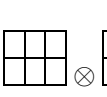
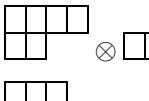
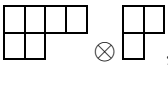
system with the open strings stretching between the giants and the operator obtained using $\mathrm{Tr}\,(A^{(2,2)})$ or $\mathrm{Tr}\,(A^{(3,3)})$ as dual to the system with one open string on each giant. In general, identify the "on the diagonal" blocks with states in which the two open strings are each on a specific giant and the "off the diagonal" blocks as states in which the open strings stretch between two giants. As a consequence of the fact that the representation R_2 appears with a multiplicity two, there is no unique way to extract two R_2 representations out of R. The specific representations obtained will depend on the details of the subgroups used in performing the restriction. These subgroups are the set of elements of the permutation group that leave an index invariant, $\sigma(i) = i$. Choosing the index to be the index of an open string, we can associate the subgroups participating with specific open strings. The subgroups are specified by dropping boxes from R, so that we can now associate boxes in R with specific open strings. This leads to a convenient graphical notation which has been developed in [30, 32]. There is an obvious generalization to the case that a representation R_1 appears n times after restricting to the subgroup.

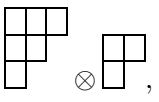
If any of the strings are identical, one needs to decompose with respect to a larger subgroup and to pick a representation for the strings which are indistinguishable. Thus, for example, if we consider a bound state of a giant system with three identical strings attached, we would consider an $S_{n-3} \otimes S_3$ subgroup of S_n. The restricted Schur polynomial would be given by $\chi^{(3)}_{R,R_1}$ with R an irrep of S_n and R_1 an irrep of $S_{n-3} \otimes S_3$. The S_3 subgroup would act by permuting the indices of the three identical strings; the S_{n-3} subgroup would act by permuting the indices of the Zs out of which the giant is composed. Write $R_1 = r_1 \times r_2$ with r_1 are irrep of S_{n-3} and r_2 an irrep of S_3. As an example, if we take R to be an irrep of S_9

$R =$  , $\qquad \dim_R = 84$

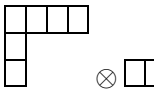
then we can have

$R_1 =$  , $\qquad \dim_{R_1} = 5,$ $\qquad R_1 =$  , $\qquad \dim_{R_1} = 10,$

$R_1 =$  , $\qquad \dim_{R_1} = 9,$ $\qquad R_1 =$  , $\qquad \dim_{R_1} = 18,$

$R_1 =$  , $\qquad \dim_{R_1} = 32,$

or

$R_1 =$  , $\qquad \dim_{R_1} = 10.$

By summing the dimensions of these representations, it is easy to see that we have indeed listed all of the representations that are subduced by R.

The giant graviton system is dual to an operator containing a product of order N fields; the open strings are dual to an operator containing a product of order $\sqrt{N}$ fields.

We have in mind the case that k is $O(1)$, n is $O(N)$ and the words $(W^{(a)})_i^j$ are a product of $O(\sqrt{N})$ fields.

We call the operator (E.1) a *restricted* Schur polynomial of representation R with representation R_1 for the restriction. We end this appendix with a summary of the graphical notation of [30], which is used heavily in this article. An operator dual to an excited giant graviton takes the form

$$\chi^{(k)}_{R,R_1}(Z,W^{(1)},\ldots,W^{(k)}) = \frac{1}{(n-k)!}\sum_{\sigma\in S_n}\mathrm{Tr}\,(\Pi\Gamma_R(\sigma))\mathrm{Tr}\,(\sigma Z^{\otimes n-k}W^{(1)}\cdots W^{(k)}),$$

where Π is a product of projection operators and/or intertwiners, used to implement the restricted trace. Π is defined by the sequence of irreducible representations used to subduce R_1 from R, as well as the chain of subgroups to which these representations belong. Since the row and column indices of the block that we trace over (denoted by R_1 in the above formula) need not coincide, we need to specify this data separately for both indices. The graphical notation summarizes this information.

For the case that we have k strings, we label the words describing the open strings $1,2,\ldots,k$. Denote the chain of subgroups involved in the reduction by $\mathcal{G}_k \subset \mathcal{G}_{k-1} \subset \cdots \subset \mathcal{G}_2 \subset \mathcal{G}_1 \subset S_n$. $\mathcal{G}_m$ is obtained by taking all elements S_n that leave the indices of the strings $W^{(i)}$ with $i \leq m$ inert. To specify the sequence of irreducible representations employed in subducing R_1, place a pair of labels into each box, a lower label and an upper label. The representations needed to subduce the row label of R_1 are obtained by starting with R. The second representation is obtained by dropping the box with upper label equal to 1; the third representation is obtained from the second by dropping the box with upper label equal to 2 and so on until the box with label k is dropped. The representations needed to subduce the column label are obtained in exactly the same way except that instead of using the upper label, we now use the lower label. For further details and explicit examples, we refer the reader to [30].

F. Boundstate of three sphere giants

In this appendix, we will compute the $+1 \to 1$ interaction for two strings attached to a bound state of three sphere giants. This example is interesting because, firstly, it does partially illustrate our claim that the methods we have developed apply to any bound state of giants and secondly, in this situation, we expect an emergent U(3) gauge theory. The three sphere giant boundstate is described by a Young diagram with three columns. When labeling the open string endpoints we will use the labels 'b', 'm' and 'l' for the first column ('b' for big brane), second column ('m' for medium brane) and third column ('l' for little brane) respectively. The relevant Cuntz chain states together with their normalizations are shown in table 2.

The labels b_0, b_1 and b_2 again determine the momenta of the giants. The giant corresponding to the first column has a momentum of $b_0 + b_1 + b_2$, the giant corresponding to the second column has a momentum of $b_0 + b_1$ and the giant corresponding to the third column has a momentum of b_0. We take b_0 to be $O(N)$ and b_1, b_2 to be $O(1)$.

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State	Normalization
$\lvert b_0, b_1-1, b_2, bb, mm\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{3b_0+2b_1+b_2+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{(b_1+b_2+1)b_1}{(b_1+b_2+2)(b_1+1)}}$
$\lvert b_0-1, b_1+1, b_2-1, bb, ll\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{3b_0+2b_1+b_2+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{(b_1+2)b_2}{(b_2+1)(b_1+1)}}$
$\lvert b_0, b_1-1, b_2, mm, bb\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{3b_0+2b_1+b_2+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{(b_1+b_2+1)b_1}{(b_1+b_2+2)(b_1+1)}}$
$\lvert b_0-1, b_1, b_2+1, mm, ll\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{3b_0+2b_1+b_2+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{(b_2+2)(b_1+b_2+3)}{(b_2+1)(b_1+b_2+2)}}$
$\lvert b_0-1, b_1+1, b_2-1, ll, bb\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{3b_0+2b_1+b_2+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{(b_1+2)b_2}{(b_2+1)(b_1+1)}}$
$\lvert b_0-1, b_1, b_2+1, ll, mm\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{3b_0+2b_1+b_2+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{(b_2+2)(b_1+b_2+3)}{(b_2+1)(b_1+b_2+2)}}$
$\lvert b_0, b_1, b_2-2, bb, bb\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{3b_0+2b_1+b_2+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{(b_2-1)(b_1+b_2)}{(b_2+1)(b_1+b_2+2)}}$
$\lvert b_0, b_1-2, b_2+2, mm, mm\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{3b_0+2b_1+b_2+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{(b_2+3)(b_1-1)}{(b_2+1)(b_1+1)}}$
$\lvert b_0-2, b_1+2, b_2, ll, ll\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{3b_0+2b_1+b_2+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{(b_1+3)(b_1+b_2+4)}{(b_1+1)(b_1+b_2+2)}}$
$\lvert b_0, b_1-1, b_2, bm, mb\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{3b_0+2b_1+b_2+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{(b_1+b_2+1)b_1}{(b_1+b_2+2)(b_1+1)}}$
$\lvert b_0, b_1-1, b_2, mb, bm\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{3b_0+2b_1+b_2+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{(b_1+b_2+1)b_1}{(b_1+b_2+2)(b_1+1)}}$
$\lvert b_0-1, b_1+1, b_2-1, bl, lb\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{3b_0+2b_1+b_2+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{(b_1+2)b_2}{(b_2+1)(b_1+1)}}$
$\lvert b_0-1, b_1+1, b_2-1, lb, bl\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{3b_0+2b_1+b_2+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{(b_1+2)b_2}{(b_2+1)(b_1+1)}}$
$\lvert b_0-1, b_1, b_2+1, ml, lm\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{3b_0+2b_1+b_2+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{(b_2+2)(b_1+b_2+3)}{(b_2+1)(b_1+b_2+2)}}$
$\lvert b_0-1, b_1, b_2+1, lm, ml\rangle$	$\left(\frac{4\pi\lambda}{N}\right)^{\frac{3b_0+2b_1+b_2+h_1+h_2-2}{2}} b_0\sqrt{f_R}\sqrt{N^{h_1+h_2-2}}\sqrt{\frac{(b_2+2)(b_1+b_2+3)}{(b_2+1)(b_1+b_2+2)}}$

Table 2: Normalization of the Cuntz states dual to a three giant system.

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To determine the boundary interactions, we start by rewriting the identities of appendix C for the case that we have a Young diagram with three columns. To obtain the boundary interaction terms in the Hamiltonian, these identities are then inverted and rewritten in terms of normalized Cuntz chain states.

The term in the Hamiltonian describing the process in which a Z hops out of the first

site of string 1 is given by

$$H_{+1\to 1}\begin{bmatrix} |b_0,b_1-1,b_2,bb,mm\rangle \\ |b_0-1,b_1+1,b_2-1,bb,ll\rangle \\ |b_0,b_1-1,b_2,mm,bb\rangle \\ |b_0-1,b_1,b_2+1,mm,ll\rangle \\ |b_0-1,b_1+1,b_2-1,ll,bb\rangle \\ |b_0-1,b_1,b_2+1,ll,mm\rangle \\ |b_0,b_1,b_2-2,bb,bb\rangle \\ |b_0,b_1-2,b_2+2,mm,mm\rangle \\ |b_0-2,b_1+2,b_2,ll,ll\rangle \\ |b_0,b_1-1,b_2,bm,mb\rangle \\ |b_0,b_1-1,b_2,mb,bm\rangle \\ |b_0-1,b_1+1,b_2-1,bl,lb\rangle \\ |b_0-1,b_1+1,b_2-1,lb,bl\rangle \\ |b_0-1,b_1,b_2+1,ml,lm\rangle \\ |b_0-1,b_1,b_2+1,lm,ml\rangle \end{bmatrix} = -\lambda\sqrt{1-\frac{b_0}{N}}M \begin{bmatrix} |b_0,b_1-1,b_2+1,bb,mm\rangle \\ |b_0-1,b_1+1,b_2,bb,ll\rangle \\ |b_0,b_1,b_2-1,mm,bb\rangle \\ |b_0-1,b_1+1,b_2,mm,ll\rangle \\ |b_0,b_1,b_2-1,ll,bb\rangle \\ |b_0,b_1-1,b_2+1,ll,mm\rangle \\ |b_0,b_1,b_2-1,bb,bb\rangle \\ |b_0,b_1-1,b_2+1,mm,mm\rangle \\ |b_0-1,b_1+1,b_2,ll,ll\rangle \\ |b_0,b_1-1,b_2+1,bm,mb\rangle \\ |b_0,b_1,b_2-1,mb,bm\rangle \\ |b_0-1,b_1+1,b_2,bl,lb\rangle \\ |b_0,b_1,b_2-1,lb,bl\rangle \\ |b_0-1,b_1+1,b_2,ml,lm\rangle \\ |b_0,b_1-1,b_2+1,lm,ml\rangle \end{bmatrix},$$

where the non-zero elements of M are given by

$$M_{1\,1} = -(b_2)_1^2(b_1+b_2)_2, \qquad M_{3\,1} = -\frac{(b_1+b_2)_2}{(b_2+1)^2(b_2+2)},$$

$$M_{6\,1} = -\frac{(b_1+2)\sqrt{b_2+2}\sqrt{b_1}}{\sqrt{b_2+1}\,(b_1+1)^{3/2}\,(b_1+b_2+2)},$$

$$M_{4\,1} = \frac{-b_1-b_2-3}{\sqrt{b_2+1}\,(b_1+1)^{3/2}\,(b_1+b_2+2)\,\sqrt{b_1}\sqrt{b_2+2}},$$

$$M_{8\,1} = -\frac{\sqrt{b_1-1}\sqrt{b_2+3}\sqrt{b_1+b_2+3}}{\sqrt{b_2+1}\sqrt{b_1+b_2+2}\,(b_2+2)\,\sqrt{b_1}}, \qquad M_{10\,1} = -\frac{(b_1+b_2)_2(b_2)_1}{(b_2+1)(b_2+2)},$$

$$M_{11\,1} = -\frac{(b_1+b_2)_2(b_2)_1}{(b_2+1)},$$

$$M_{14\,1} = \frac{\sqrt{b_2+2}\sqrt{b_1+2}}{(b_1+1)^{3/2}\,\sqrt{b_2+1}\,(b_1+b_2+2)},$$

$$M_{15\,1} = \frac{(b_1+b_2+3)\,\sqrt{b_1+2}}{(b_1+1)^{3/2}\,\sqrt{b_2+1}\,(b_1+b_2+2)\,\sqrt{b_2+2}}, \qquad M_{2\,2} = -(b_1+b_2)_2^2(b_2)_1,$$

$$M_{5\,2} = \frac{(b_2)_1}{(b_1+b_2+2)^2(b_1+b_2+3)},$$

$$M_{4\,2} = -\frac{b_1\,\sqrt{b_1+b_2+3}\sqrt{b_1+2}}{(b_1+1)^{3/2}\,(b_2+1)\,\sqrt{b_1+b_2+2}},$$

$$M_{6\,2} = \frac{-b_2-2}{(b_2+1)\,(b_1+1)^{3/2}\,\sqrt{b_1+b_2+2}\sqrt{b_1+2}\sqrt{b_1+b_2+3}}, \qquad M_{13\,2} = -\frac{(b_2)_1(b_1+b_2)_2}{(b_1+b_2+2)},$$

$$M_{9\,2} = -\frac{\sqrt{b_1+b_2+4}\sqrt{b_1+3}\sqrt{b_2+2}}{\sqrt{b_2+1}\sqrt{b_1+b_2+2}\sqrt{b_1+2}\,(b_1+b_2+3)},$$

$$M_{12\,2} = -\frac{(b_2)_1(b_1+b_2)_2}{(b_1+b_2+2)(b_1+b_2+3)}$$

$$M_{14\,2} = -\frac{(b_2+2)\sqrt{b_1}}{(b_1+1)^{3/2}(b_2+1)\sqrt{b_1+b_2+2}\sqrt{b_1+b_2+3}}, \qquad M_{1\,3} = \frac{(b_1)_1}{b_2(b_2+1)^2},$$

$$M_{15\,2} = -\frac{\sqrt{b_1+b_2+3}\sqrt{b_1}}{(b_1+1)^{3/2}(b_2+1)\sqrt{b_1+b_2+2}}, \qquad M_{3\,3} = -(b_2)_1^2(b_1)_1,$$

$$M_{2\,3} = \frac{b_1+2}{\sqrt{b_2+1}\,(b_1+1)\,(b_1+b_2+2)^{3/2}\sqrt{b_1+b_2+1}\sqrt{b_2}}, \qquad M_{10\,3} = \frac{(b_1)_1(b_2)_1}{(b_2+1)},$$

$$M_{5\,3} = -\frac{(b_1+b_2+3)\sqrt{b_2}\sqrt{b_1+b_2+1}}{\sqrt{b_2+1}\,(b_1+1)\,(b_1+b_2+2)^{3/2}}, \qquad M_{11\,3} = \frac{(b_1)_1(b_2)_1}{b_2(b_2+1)},$$

$$M_{7\,3} = \frac{\sqrt{b_2-1}\sqrt{b_1+b_2}\sqrt{b_1+2}}{\sqrt{b_2+1}\sqrt{b_1+1}b_2\sqrt{b_1+b_2+1}},$$

$$M_{12\,3} = \frac{\sqrt{b_2}\sqrt{b_1+b_2+3}}{(b_1+b_2+2)^{3/2}(b_1+1)\sqrt{b_2+1}},$$

$$M_{13\,3} = -\frac{(b_1+2)\sqrt{b_1+b_2+3}}{(b_1+b_2+2)^{3/2}(b_1+1)\sqrt{b_2+1}\sqrt{b_2}},$$

$$M_{2\,4} = \frac{\sqrt{b_1+2}\,(b_1+b_2+1)\sqrt{b_1+b_2+3}}{(b_2+1)\sqrt{b_1+1}\,(b_1+b_2+2)^{3/2}}, \qquad M_{4\,4} = -(b_1)_1^2(b_2)_1,$$

$$M_{5\,4} = -\frac{b_2}{\sqrt{b_1+1}\,(b_2+1)\,(b_1+b_2+2)^{3/2}\sqrt{b_1+b_2+3}\sqrt{b_1+2}},$$

$$M_{6\,4} = -\frac{\sqrt{b_2+2}\sqrt{b_2}}{(b_1+1)^2(b_2+1)(b_1+2)},$$

$$M_{9\,4} = -\frac{\sqrt{b_1+b_2+4}\sqrt{b_1+3}\sqrt{b_2}}{\sqrt{b_2+1}\sqrt{b_1+1}\,(b_1+2)\sqrt{b_1+b_2+3}},$$

$$M_{12\,4} = -\frac{b_2\sqrt{b_1+b_2+1}}{(b_1+b_2+2)^{3/2}(b_2+1)\sqrt{b_1+1}\sqrt{b_1+2}},$$

$$M_{13\,4} = \frac{\sqrt{b_1+2}\sqrt{b_1+b_2+1}}{(b_1+b_2+2)^{3/2}(b_2+1)\sqrt{b_1+1}}, \qquad M_{14\,4} = -\frac{(b_2)_1(b_1)_1}{(b_1+1)(b_1+2)},$$

$$M_{1\,5} = \frac{b_1}{(b_2+1)^{3/2}(b_1+1)\sqrt{b_1+b_2+2}\sqrt{b_2}\sqrt{b_1+b_2+1}}, \qquad M_{15\,4} = -\frac{(b_2)_1(b_1)_1}{(b_1+1)},$$

$$M_{2\,5} = \frac{(b_1)_1}{(b_1+b_2+2)^2(b_1+b_2+1)},$$

$$M_{3\,5} = \frac{\sqrt{b_1+b_2+1}\,(b_2+2)\sqrt{b_2}}{(b_2+1)^{3/2}(b_1+1)\sqrt{b_1+b_2+2}}, \qquad M_{5\,5} = -(b_1)_1(b_1+b_2)_2^2,$$

$$M_{7\,5} = \frac{\sqrt{b_2-1}\sqrt{b_1+b_2}\sqrt{b_1}}{\sqrt{b_1+1}\sqrt{b_1+b_2+2}\,(b_1+b_2+1)\sqrt{b_2}},$$

$$M_{10\,5} = -\frac{\sqrt{b_1+b_2+1}\sqrt{b_2+2}}{(b_2+1)^{3/2}(b_1+1)\sqrt{b_1+b_2+2}},$$

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$$M_{11\,5} = -\frac{b_1\sqrt{b_2+2}}{(b_2+1)^{3/2}(b_1+1)\sqrt{b_1+b_2+2}\sqrt{b_1+b_2+1}}, \qquad M_{12\,5} = \frac{(b_1)_1(b_1+b_2)_2}{b_1+b_2+2},$$

$$M_{13\,5} = -\frac{(b_1)_1(b_1+b_2)_2}{(b_1+b_2+2)(b_1+b_2+1)},$$

$$M_{1\,6} = \frac{\sqrt{b_1}b_2\sqrt{b_2+2}}{\sqrt{b_1+1}(b_2+1)^{3/2}(b_1+b_2+2)},$$

$$M_{3\,6} = \frac{b_1+b_2+1}{\sqrt{b_1+1}(b_2+1)^{3/2}(b_1+b_2+2)\sqrt{b_2+2}\sqrt{b_1}}, \qquad M_{4\,6} = \frac{(b_1+b_2)_2}{b_1(b_1+1)^2},$$

$$M_{6\,6} = -(b_1)_1^2(b_1+b_2)_2,$$

$$M_{8\,6} = \frac{\sqrt{b_1-1}\sqrt{b_2+3}\sqrt{b_1+b_2+1}}{\sqrt{b_1+1}\sqrt{b_1+b_2+2}\sqrt{b_2+2}b_1},$$

$$M_{10\,6} = \frac{(b_1+b_2+1)\sqrt{b_2}}{(b_2+1)^{3/2}\sqrt{b_1+1}(b_1+b_2+2)\sqrt{b_1}},$$

$$M_{11\,6} = \frac{\sqrt{b_1}\sqrt{b_2}}{(b_2+1)^{3/2}\sqrt{b_1+1}(b_1+b_2+2)}, \qquad M_{14\,6} = \frac{(b_1)_1(b_1+b_2)_2}{b_1+1},$$

$$M_{1\,7} = -\frac{\sqrt{b_1}\sqrt{b_2+2}\sqrt{b_1+b_2+3}}{(b_2+1)^2\sqrt{b_1+1}\sqrt{b_1+b_2+2}\sqrt{b_2}}, \qquad M_{15\,6} = -\frac{(b_1)_1(b_1+b_2)_2}{b_1(b_1+1)},$$

$$M_{2\,7} = -\frac{\sqrt{b_1+2}\sqrt{b_2+2}\sqrt{b_1+b_2+3}}{\sqrt{b_2+1}\sqrt{b_1+1}(b_1+b_2+2)^2\sqrt{b_1+b_2+1}},$$

$$M_{3\,7} = -\frac{\sqrt{b_1}\sqrt{b_2+2}\sqrt{b_1+b_2+3}\sqrt{b_2}}{\sqrt{b_1+1}(b_2+1)^2\sqrt{b_1+b_2+2}}, \qquad M_{5\,7} = -\frac{\sqrt{b_1+2}\sqrt{b_2+2}(b_1+b_2)_2}{\sqrt{b_1+1}\sqrt{b_2+1}(b_1+b_2+2)},$$

$$M_{7\,7} = -\frac{\sqrt{b_1+b_2+3}\sqrt{b_2+2}\sqrt{b_2-1}\sqrt{b_1+b_2}}{\sqrt{b_2}\sqrt{b_1+b_2+1}\sqrt{b_2+1}\sqrt{b_1+b_2+2}}, \qquad M_{10\,7} = \frac{\sqrt{b_1}\sqrt{b_1+b_2+3}}{(b_2+1)^2\sqrt{b_1+1}\sqrt{b_1+b_2+2}},$$

$$M_{11\,7} = \frac{\sqrt{b_1}(b_2+2)\sqrt{b_1+b_2+3}}{(b_2+1)^2\sqrt{b_1+1}\sqrt{b_1+b_2+2}}, \qquad M_{12\,7} = \frac{\sqrt{b_1+2}\sqrt{b_2+2}}{(b_1+b_2+2)^2\sqrt{b_2+1}\sqrt{b_1+1}},$$

$$M_{13\,7} = \frac{\sqrt{b_1+2}\sqrt{b_2+2}(b_1+b_2+3)}{(b_1+b_2+2)^2\sqrt{b_2+1}\sqrt{b_1+1}}, \qquad M_{1\,8} = \frac{\sqrt{b_1+2}\sqrt{b_1+b_2+1}(b_2)_1}{(b_2+1)\sqrt{b_1+1}\sqrt{b_1+b_2+2}},$$

$$M_{3\,8} = -\frac{\sqrt{b_1+2}\sqrt{b_1+b_2+1}\sqrt{b_2}}{(b_2+1)^2\sqrt{b_1+1}\sqrt{b_1+b_2+2}\sqrt{b_2+2}},$$

$$M_{4\,8} = -\frac{\sqrt{b_1+2}\sqrt{b_1+b_2+3}\sqrt{b_2}}{(b_1+1)^2\sqrt{b_2+1}\sqrt{b_1+b_2+2}\sqrt{b_1}},$$

$$M_{6\,8} = -\frac{\sqrt{b_1+2}\sqrt{b_1+b_2+3}\sqrt{b_2}\sqrt{b_1}}{\sqrt{b_2+1}(b_1+1)^2\sqrt{b_1+b_2+2}}, \qquad M_{8\,8} = -\frac{\sqrt{b_1-1}\sqrt{b_2+3}\sqrt{b_1+2}\sqrt{b_2}}{\sqrt{b_1+1}\sqrt{b_2+1}\sqrt{b_1}\sqrt{b_2+2}},$$

$$M_{10\,8} = -\frac{\sqrt{b_1+2}\sqrt{b_1+b_2+1}b_2}{(b_2+1)^2\sqrt{b_1+1}\sqrt{b_1+b_2+2}}, \qquad M_{11\,8} = \frac{\sqrt{b_1+2}\sqrt{b_1+b_2+1}}{(b_2+1)^2\sqrt{b_1+1}\sqrt{b_1+b_2+2}},$$

$$M_{14\,8} = \frac{\sqrt{b_1+b_2+3}\sqrt{b_2}}{(b_1+1)^2\sqrt{b_2+1}\sqrt{b_1+b_2+2}}, \qquad M_{15\,8} = \frac{(b_1+2)\sqrt{b_1+b_2+3}\sqrt{b_2}}{(b_1+1)^2\sqrt{b_2+1}\sqrt{b_1+b_2+2}},$$

$$M_{2\,9} = \frac{\sqrt{b_1}\sqrt{b_2}(b_1+b_2)_2}{\sqrt{b_1+1}\sqrt{b_2+1}(b_1+b_2+2)},$$

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$$M_{4\,9}=\frac{\sqrt{b_1}\sqrt{b_1+b_2+1}\sqrt{b_2+2}\sqrt{b_1+2}}{\sqrt{b_1+b_2+2}\sqrt{b_2+1}\,(b_1+1)^2}$$

$$M_{5\,9}=-\frac{\sqrt{b_1}\sqrt{b_1+b_2+1}\sqrt{b_2}}{\sqrt{b_1+1}\sqrt{b_2+1}\,(b_1+b_2+2)^2\,\sqrt{b_1+b_2+3}},$$

$$M_{6\,9}=-\frac{\sqrt{b_1}\sqrt{b_1+b_2+1}\sqrt{b_2+2}}{\sqrt{b_2+1}\,(b_1+1)^2\,\sqrt{b_1+b_2+2}\sqrt{b_1+2}},$$

$$M_{9\,9}=-\frac{\sqrt{b_1+b_2+4}\sqrt{b_1+3}\sqrt{b_1}\sqrt{b_1+b_2+1}}{\sqrt{b_1+1}\sqrt{b_1+b_2+2}\sqrt{b_1+b_2+3}\sqrt{b_1+2}},$$

$$M_{12\,9}=-\frac{\sqrt{b_1}\,(b_1+b_2+1)\,\sqrt{b_2}}{(b_1+b_2+2)^2\,\sqrt{b_1+1}\sqrt{b_2+1}},$$

$$M_{13\,9}=\frac{\sqrt{b_1}\sqrt{b_2}}{(b_1+b_2+2)^2\,\sqrt{b_1+1}\sqrt{b_2+1}},$$

$$M_{14\,9}=-\frac{b_1\,\sqrt{b_1+b_2+1}\sqrt{b_2+2}}{(b_1+1)^2\sqrt{b_2+1}\sqrt{b_1+b_2+2}},$$

$$M_{15\,9}=\frac{\sqrt{b_1+b_2+1}\sqrt{b_2+2}}{(b_1+1)^2\,\sqrt{b_2+1}\sqrt{b_1+b_2+2}},$$

$$M_{3\,10}=-\frac{\sqrt{b_2+3}(b_1+b_2)_2}{(b_2+2)\sqrt{b_2+1}},$$

$$M_{4\,10}=\frac{\sqrt{b_2+3}}{(b_1+b_2+2)\,\sqrt{b_2+2}\sqrt{b_1}\sqrt{b_1+1}},$$

$$M_{8\,10}=\frac{\sqrt{b_1-1}\sqrt{b_1+b_2+3}}{\sqrt{b_1+b_2+2}\,(b_2+2)\,\sqrt{b_1}},$$

$$M_{10\,10}=-\frac{\sqrt{b_2+3}\sqrt{b_1+b_2+1}\sqrt{b_1+b_2+3}\sqrt{b_2}}{\sqrt{b_2+1}\sqrt{b_2+2}\,(b_1+b_2+2)},$$

$$M_{15\,10}=-\frac{\sqrt{b_2+3}\sqrt{b_1+2}}{\sqrt{b_1+1}\,(b_1+b_2+2)\,\sqrt{b_2+2}},$$

$$M_{1\,11}=\frac{(b_2-1)\,\sqrt{b_1}\sqrt{b_1+2}}{(b_1+1)\,\sqrt{{b_2}^2-1}b_2},$$

$$M_{2\,11}=\frac{\sqrt{b_2-1}}{(b_1+1)\,\sqrt{b_2}\sqrt{b_1+b_2+1}\sqrt{b_1+b_2+2}},$$

$$M_{7\,11}=-\frac{\sqrt{b_1+b_2}\sqrt{b_1+2}}{\sqrt{b_1+1}b_2\,\sqrt{b_1+b_2+1}},$$

$$M_{11\,11}=-\frac{\sqrt{b_2-1}\sqrt{b_1}\sqrt{b_1+2}\sqrt{b_2+2}}{\sqrt{b_2+1}\sqrt{b_2}\,(b_1+1)},$$

$$M_{13\,11}=-\frac{\sqrt{b_2-1}\sqrt{b_1+b_2+3}}{\sqrt{b_1+b_2+2}\,(b_1+1)\,\sqrt{b_2}},$$

$$M_{5\,12}=-\frac{\sqrt{b_1+b_2+4}\sqrt{b_2+2}\sqrt{b_2}}{(b_2+1)\sqrt{b_1+b_2+2}\,(b_1+b_2+3)},$$

$$M_{6\,12}=-\frac{\sqrt{b_1+b_2+4}}{(b_2+1)\,\sqrt{b_1+b_2+3}\sqrt{b_1+2}\sqrt{b_1+1}},$$

$$M_{9\,12}=\frac{\sqrt{b_1+3}\sqrt{b_2+2}}{\sqrt{b_2+1}\,(b_1+b_2+3)\,\sqrt{b_1+2}},$$

$$M_{12\,12}=-\frac{\sqrt{b_1+b_2+4}\sqrt{b_2+2}\sqrt{b_2}\sqrt{b_1+b_2+1}}{\sqrt{b_1+b_2+2}\sqrt{b_1+b_2+3}\,(b_2+1)},$$

$$M_{14\,12}=-\frac{\sqrt{b_1+b_2+4}\sqrt{b_1}}{\sqrt{b_1+1}\,(b_2+1)\,\sqrt{b_1+b_2+3}},$$

$$M_{1\,13}=-\frac{\sqrt{b_1+b_2}}{(b_1+1)\,\sqrt{b_1+b_2+1}\sqrt{b_2}\sqrt{b_2+1}},$$

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$$M_{2\,13} = \frac{\sqrt{b_1+b_2}\sqrt{b_1}\sqrt{b_1+2}}{(b_1+1)\sqrt{b_1+b_2+2}\,(b_1+b_2+1)},$$

$$M_{7\,13} = -\frac{\sqrt{b_2-1}\sqrt{b_1}}{\sqrt{b_1+1}\,(b_1+b_2+1)\,\sqrt{b_2}}, \qquad M_{11\,13} = \frac{\sqrt{b_1+b_2}\sqrt{b_2+2}}{\sqrt{b_2+1}\,(b_1+1)\,\sqrt{b_1+b_2+1}},$$

$$M_{13\,13} = -\frac{\sqrt{b_1+b_2}\sqrt{b_1}\sqrt{b_1+2}\sqrt{b_1+b_2+3}}{\sqrt{b_1+b_2+1}\sqrt{b_1+b_2+2}\,(b_1+1)},$$

$$M_{5\,14} = \frac{\sqrt{b_1+3}}{(b_2+1)\,\sqrt{b_1+2}\sqrt{b_1+b_2+3}\sqrt{b_1+b_2+2}},$$

$$M_{6\,14} = -\frac{(b_1+3)\,\sqrt{b_2+2}\sqrt{b_2}}{(b_2+1)\,\sqrt{{b_1}^2+4\,b_1+3}\,(b_1+2)}, \qquad M_{9\,14} = \frac{\sqrt{b_1+b_2+4}\sqrt{b_2}}{\sqrt{b_2+1}\,(b_1+2)\,\sqrt{b_1+b_2+3}},$$

$$M_{12\,14} = \frac{\sqrt{b_1+3}\sqrt{b_1+b_2+1}}{\sqrt{b_1+b_2+2}\,(b_2+1)\,\sqrt{b_1+2}}, \qquad M_{14\,14} = -\frac{\sqrt{b_1+3}\sqrt{b_2+2}\sqrt{b_2}\sqrt{b_1}}{\sqrt{b_1+1}\sqrt{b_1+2}\,(b_2+1)},$$

$$M_{3\,15} = \frac{\sqrt{b_1-1}}{(b_1+b_2+2)\,\sqrt{b_1}\sqrt{b_2+2}\sqrt{b_2+1}}, \qquad M_{4\,15} = \frac{(b_1-1)\,(b_1+b_2)_2}{\sqrt{{b_1}^2-1}b_1},$$

$$M_{8\,15} = -\frac{\sqrt{b_2+3}\sqrt{b_1+b_2+1}}{\sqrt{b_1+b_2+2}b_1\,\sqrt{b_2+2}}, \qquad M_{10\,15} = \frac{\sqrt{b_1-1}\sqrt{b_2}}{\sqrt{b_2+1}\,(b_1+b_2+2)\,\sqrt{b_1}},$$

$$M_{15\,15} = -\frac{\sqrt{b_1-1}\sqrt{b_1+b_2+1}\sqrt{b_1+b_2+3}\sqrt{b_1+2}}{\sqrt{b_1+1}\sqrt{b_1}\,(b_1+b_2+2)}.$$

For large b_1 and b_2, we find that $M = -\mathbf{1}$ with $\mathbf{1}$ the 15×15 identity matrix. We can also identify terms in M that behave as b_1^{-1}

$$M_1 = \frac{1}{b_1}\begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&1\\
0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&0&-1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&-1\\
0&0&0&-1&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0\\
0&0&0&0&0&1&0&0&-1&0&0&0&0&0&0\\
0&0&0&-1&0&0&0&1&0&0&0&0&0&0&0
\end{bmatrix},$$

terms that behave as b_2^{-1}

$$M_2 = \frac{1}{b_2}\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

and terms that behave as $(b_1 + b_2)^{-1}$

$$M_3 = \frac{1}{b_1 + b_2}\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.$$

By looking at the Cuntz chain states, it is straight forward to see that M_1 is reproduced by ribbon diagrams in which a pair of labels undergoes a $l \leftrightarrow m$ transition, that M_2 is reproduced by ribbon diagrams in which a pair of labels undergoes a $b \leftrightarrow m$ transition and that M_3 is reproduced by ribbon diagrams in which a pair of labels undergoes a $l \leftrightarrow b$ transition. This is exactly the structure expected from an emergent U(3) gauge theory.

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[4] J.P. Rodrigues, *Large-N spectrum of two matrices in a harmonic potential and BMN energies*, *JHEP* **12** (2005) 043 [hep-th/0510244];
A. Donos, A. Jevicki and J.P. Rodrigues, *Matrix model maps in AdS/CFT*, *Phys. Rev.* **D 72** (2005) 125009 [hep-th/0507124];
R. de Mello Koch, A. Jevicki and J. P. Rodrigues, *Instantons in $c = 0$ CSFT*, *JHEP* **04** (2005) 011 [hep-th/0412319];
R. de Mello Koch, A. Donos, A. Jevicki and J.P. Rodrigues, *Derivation of string field theory from the large N BMN limit*, *Phys. Rev.* **D 68** (2003) 065012 [hep-th/0305042];
R. de Mello Koch, A. Jevicki and J.P. Rodrigues, *Collective string field theory of matrix models in the BMN limit*, *Int. J. Mod. Phys.* **A 19** (2004) 1747 [hep-th/0209155].

[5] D. Berenstein and R. Cotta, *A Monte-Carlo study of the AdS/CFT correspondence: an exploration of quantum gravity effects*, *JHEP* **04** (2007) 071 [hep-th/0702090]; *Aspects of emergent geometry in the AdS/CFT context*, *Phys. Rev.* **D 74** (2006) 026006 [hep-th/0605220];
D. Berenstein and D.H. Correa, *Emergent geometry from q-deformations of $N = 4$ super Yang-Mills*, *JHEP* **08** (2006) 006 [hep-th/0511104];
D. Berenstein, *Large N BPS states and emergent quantum gravity*, *JHEP* **01** (2006) 125 [hep-th/0507203]; *Strings on conifolds from strong coupling dynamics, part I*, arXiv:0710.2086.

[6] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111].

[7] D. Berenstein, *Shape and holography: studies of dual operators to giant gravitons*, *Nucl. Phys.* **B 675** (2003) 179 [hep-th/0306090].

[8] D.E. Berenstein, J.M. Maldacena and H.S. Nastase, *Strings in flat space and pp waves from $N = 4$ super Yang-Mills*, *JHEP* **04** (2002) 013 [hep-th/0202021].

[9] R. Russo and A. Tanzini, *The duality between IIB string theory on pp-wave and $N = 4$ sym: a status report*, *Class. and Quant. Grav.* **21** (2004) S1265 [hep-th/0401155];
D. Sadri and M.M. Sheikh-Jabbari, *The plane-wave/super Yang-Mills duality*, *Rev. Mod. Phys.* **76** (2004) 853 [hep-th/0310119].

[10] J. McGreevy, L. Susskind and N. Toumbas, *Invasion of the giant gravitons from anti-de Sitter space*, *JHEP* **06** (2000) 008 [hep-th/0003075].

[11] M.T. Grisaru, R.C. Myers and O. Tafjord, *SUSY and Goliath*, *JHEP* **08** (2000) 040 [hep-th/0008015].

[12] A. Hashimoto, S. Hirano and N. Itzhaki, *Large branes in AdS and their field theory dual*, *JHEP* **08** (2000) 051 [hep-th/0008016].

[13] S.R. Das, A. Jevicki and S.D. Mathur, *Giant gravitons, BPS bounds and noncommutativity*, *Phys. Rev.* **D 63** (2001) 044001 [hep-th/0008088]; *Vibration modes of giant gravitons*, *Phys. Rev.* **D 63** (2001) 024013 [hep-th/0009019].

[14] S. Corley, A. Jevicki and S. Ramgoolam, *Exact correlators of giant gravitons from dual $N = 4$ SYM theory*, *Adv. Theor. Math. Phys.* **5** (2002) 809 [hep-th/0111222].

JHEP02(2008)029

[15] V. Balasubramanian, M. Berkooz, A. Naqvi and M.J. Strassler, *Giant gravitons in conformal field theory*, *JHEP* **04** (2002) 034 [hep-th/0107119].

[16] V. Balasubramanian, M.-x. Huang, T.S. Levi and A. Naqvi, *Open strings from $N = 4$ super Yang-Mills*, *JHEP* **08** (2002) 037 [hep-th/0204196];
O. Aharony, Y.E. Antebi, M. Berkooz and R. Fishman, *Holey sheets: Pfaffians and subdeterminants as D-brane operators in large N gauge theories*, *JHEP* **12** (2002) 096 [hep-th/0211152];
D. Berenstein, *Shape and holography: studies of dual operators to giant gravitons*, *Nucl. Phys.* **B 675** (2003) 179 [hep-th/0306090].

[17] D. Berenstein, D.H. Correa and S.E. Vazquez, *A study of open strings ending on giant gravitons, spin chains and integrability*, *JHEP* **09** (2006) 065 [hep-th/0604123].

[18] D. Berenstein and S.E. Vazquez, *Integrable open spin chains from giant gravitons*, *JHEP* **06** (2005) 059 [hep-th/0501078];
D. Berenstein, D.H. Correa and S.E. Vazquez, *Quantizing open spin chains with variable length: an example from giant gravitons*, *Phys. Rev. Lett.* **95** (2005) 191601 [hep-th/0502172];
R. de Mello Koch, N. Ives, J. Smolic and M. Smolic, *Unstable giants*, *Phys. Rev.* **D 73** (2006) 064007 [hep-th/0509007];
A. Agarwal, *Open spin chains in super Yang-Mills at higher loops: Some potential problems with integrability*, hep-th/0603067;
K. Okamura and K. Yoshida, *Higher loop Bethe ansatz for open spin-chains in AdS/CFT*, hep-th/0604100;
D.H. Correa and G.A. Silva, *Dilatation operator and the super Yang-Mills duals of open strings on AdS Giant Gravitons*, hep-th/0608128;
N. Mann and S.E. Vazquez, *Classical open string integrability*, hep-th/0612038.

[19] M. Ali-Akbari, M.M. Sheikh-Jabbari and M. Torabian, *Tiny graviton matrix theory/SYM correspondence: analysis of BPS states*, *Phys. Rev.* **D 74** (2006) 066005 [hep-th/0606117];
M.M. Sheikh-Jabbari and M. Torabian, *Classification of all 1/2 BPS solutions of the tiny graviton matrix theory*, *JHEP* **04** (2005) 001 [hep-th/0501001];
M.M. Sheikh-Jabbari, *Tiny graviton matrix theory: DLCQ of IIB plane-wave string theory, a conjecture*, *JHEP* **09** (2004) 017 [hep-th/0406214].

[20] V. Balasubramanian, D. Berenstein, B. Feng and M.-x. Huang, *D-branes in Yang-Mills theory and emergent gauge symmetry*, *JHEP* **03** (2005) 006 [hep-th/0411205].

[21] V. Balasubramanian, J. de Boer, V. Jejjala and J. Simon, *Entropy of near-extremal black holes in AdS_5*, arXiv:0707.3601.

[22] J.A. Minahan and K. Zarembo, *The Bethe-ansatz for $N = 4$ super Yang-Mills*, *JHEP* **03** (2003) 013 [hep-th/0212208].

[23] N. Beisert and M. Staudacher, *The $N = 4$ SYM integrable super spin chain*, *Nucl. Phys.* **B 670** (2003) 439 [hep-th/0307042].

[24] D. Berenstein and S.E. Vazquez, *Integrable open spin chains from giant gravitons*, *JHEP* **06** (2005) 059 [hep-th/0501078].

[25] D. Berenstein, D.H. Correa and S.E. Vazquez, *Quantizing open spin chains with variable length: an example from giant gravitons*, *Phys. Rev. Lett.* **95** (2005) 191601 [hep-th/0502172].

JHEP02(2008)029

[26] M. Kruczenski, *Spin chains and string theory*, *Phys. Rev. Lett.* **93** (2004) 161602 [hep-th/0311203].

[27] D.H. Correa and G.A. Silva, *Dilatation operator and the super Yang-Mills duals of open strings on AdS giant gravitons*, *JHEP* **11** (2006) 059 [hep-th/0608128].

[28] R. de Mello Koch, N. Ives, J. Smolic and M. Smolic, *Unstable giants*, *Phys. Rev.* **D 73** (2006) 064007 [hep-th/0509007].

[29] D.M. Hofman and J.M. Maldacena, *Reflecting magnons*, *JHEP* **11** (2007) 063 [arXiv:0708.2272].

[30] R. de Mello Koch, J. Smolic and M. Smolic, *Giant gravitons - with strings attached (I)*, *JHEP* **06** (2007) 074 [hep-th/0701066].

[31] S. Corley and S. Ramgoolam, *Finite factorization equations and sum rules for BPS correlators in $N = 4$ SYM theory*, *Nucl. Phys.* **B 641** (2002) 131 [hep-th/0205221];
R. de Mello Koch and R. Gwyn, *Giant graviton correlators from dual* SU(N) *super Yang-Mills theory*, *JHEP* **11** (2004) 081 [hep-th/0410236];
T.W. Brown, *Half-BPS* SU(N) *correlators in $N = 4$ SYM*, hep-th/0703202;
S. Ramgoolam, *Wilson loops in* 2*D Yang-Mills: Euler characters and loop equations*, *Int. J. Mod. Phys.* **A 11** (1996) 3885 [hep-th/9412110].

[32] R. de Mello Koch, J. Smolic and M. Smolic, *Giant gravitons - with strings attached (II)*, *JHEP* **09** (2007) 049 [hep-th/0701067].

[33] H. Lin, O. Lunin and J.M. Maldacena, *Bubbling AdS space and* 1/2 *BPS geometries*, *JHEP* **10** (2004) 025 [hep-th/0409174];
H.Y. Chen, D.H. Correa and G.A. Silva, *Geometry and topology of bubble solutions from gauge theory*, *Phys. Rev.* **D 76** (2007) 026003 [hep-th/0703068].

[34] R. de Mello Koch and M. Stephanou, *LLM geometries from Young diagrams*, work in progress.

[35] E. D'Hoker, D.Z. Freedman and W. Skiba, *Field theory tests for correlators in the AdS/CFT correspondence*, *Phys. Rev.* **D 59** (1999) 045008 [hep-th/9807098];
W. Skiba, *Correlators of short multi-trace operators in $N = 4$ supersymmetric Yang-Mills*, *Phys. Rev.* **D 60** (1999) 105038 [hep-th/9907088];
N.R. Constable, D.Z. Freedman, M. Headrick, S. Minwalla, L. Motl, A. Postnikov and W. Skiba, *PP-wave string interactions from perturbative Yang-Mills theory*, *JHEP* **07** (2002) 017 [hep-th/0205089].

[36] V. Balasubramanian and A. Naqvi, *Giant gravitons and a correspondence principle*, *Phys. Lett.* **B 528** (2002) 111 [hep-th/0111163].

[37] S. Hirano, *Fat magnon*, *JHEP* **04** (2007) 010 [hep-th/0610027].

[38] R.C. Myers, *Dielectric-branes*, *JHEP* **12** (1999) 022 [hep-th/9910053].

[39] Y. Kimura and S. Ramgoolam, *Branes, anti-branes and Brauer algebras in gauge-gravity duality*, *JHEP* **11** (2007) 078 [arXiv:0709.2158].

[40] J.-Q. Chen, *Group Representation Theory for Physicists*, World Scientific, Singapore (1989).

JHEP02(2008)029